

Content and Language Objective:

Students will discover the relationship between rational exponents and roots and be able to write and solve problems involving rational exponents and roots.

Warm Up

1. $8^x = 4$

$(2^3)^x = 2^2$

$\frac{3x}{3} = \frac{2}{3}$
 $x = \frac{2}{3}$

2. $27^x = \frac{1}{81}$

$(3^3)^x = \frac{1}{3^4}$

$(3^3)^x = 3^{-4}$
 $3x = -4$
 $x = -\frac{4}{3}$

3. $x^4 = 3000$

$\sqrt[4]{x^4} = \sqrt[4]{3000}$
 $x = 7.40$

$x^{4/1} = 3000$
 $x^{1/1} = 3000^{1/4}$
 $x = 7.40$

4. $\frac{6x^{2.5}}{6} = \frac{90}{6}$
 $x^{2.5} = 15$

$x = 15^{1/2.5}$
 $x = 2.95$

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DEFINITION OF RATIONAL EXPONENTS

The power of a power property shows that $a^{m/n} = (a^{1/n})^m$ and $a^{m/n} = (a^m)^{1/n}$, so

$$a^{\frac{m}{n}} = (\sqrt[n]{a})^m \text{ or } \sqrt[n]{a^m} \text{ for } a > 0$$

What does this all mean?????

This means that when we have a fraction as our exponent, a rational exponent, we can rewrite our equation as a root.

So rational exponents are shortcuts for writing a root.

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In order to have a better understanding of the relationship between roots and rational exponents, let's work through some problems that are part of this situation.

Example #1

$$\left(\sqrt[4]{a}\right)^4 = (14)^4$$
$$a = 38416$$

$$\sqrt[4]{a} = 14$$
$$a^{1/4} = 14$$
$$\cancel{a^{1/4}} = 14^{4/1}$$
$$a = 14^4$$
$$a = 38416$$

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Example #2

$$\sqrt[9]{b^5} = 26$$

$$b^{5/9} = 26$$

$$b^{5/9 \cdot 9/5} = 26^{9/5}$$

$$b = 352.3$$

Example #3

$$(\sqrt[3]{c})^8 = 47$$

$$\sqrt[3]{c^8} = 47$$

$$c^{8/3} = 47$$

$$c = 47^{3/8}$$

$$c = 4.2$$

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Example #4

$$\sqrt[6]{a} = 4.2$$

$$(\sqrt[6]{a})^6 = 4.2^6$$

$$a = 4.2^6$$

$$a = 5489.03$$

Example #5

$$\sqrt[10]{b^8} = 14.3$$

$$b^{8 \cdot \frac{10}{8}} = 14.3^{\frac{10}{8}}$$

$$b = 27.8$$

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Example #6

$$\frac{1}{\sqrt[2]{c}} = 0.55$$

$$\frac{1}{c^{1/2}} = 0.55$$

$$c^{-1/2} = 0.55$$

$$c = (0.55)^{-2} = 3.3$$

Example #7

$$\sqrt[5]{4w^3}$$

$$\begin{aligned}\sqrt[5]{4^1 w^3} &= 4^{1/5} w^{3/5} \\ &= 1.3 w^{3/5}\end{aligned}$$

