

KEY POINTS

Section 1.3 Equivalent Expressions

- When are expressions equivalent?
- Evaluating expressions to see when they are equal
- Constructing expressions

Section 1.4 Equivalent Equations

- When are equations equivalent?
- Valid operations on equations
- Isolating variables
- The difference between equivalent equations and equivalent expressions.

Warm - Up

Section 1.3
Equivalent
Expressions

Write about how expressions and equations are similar and different.

Share Out

Section 1.3
Equivalent
Expressions

Similarities

Both have variables
Both have numbers

Differences

Equations have = signs
Expressions don't have =
Expressions are evaluated
Equations are solved

Examples

Section 1.3

Equivalent Expressions

Example #1

You are given two expressions that you have to determine if they are equal or not.

$$\frac{t}{2} = \frac{2}{2} = ① \quad \frac{1}{2}t = \frac{1}{2} \cdot \frac{2}{1} = \frac{2}{2} = ①$$

How can we determine if these two expressions are equal?

Substitute the same value into both expressions

$$t = 2$$

Examples

Using what we know determine if the following expressions are equivalent or not.

$$x=2 \quad y=2$$

$$\sqrt{x+y}$$

\neq

$$\sqrt{x} + \sqrt{y}$$

$$\sqrt{2} + \sqrt{2}$$

$$2\sqrt{2}$$

$$\sqrt{2+2}$$

$$\sqrt{4} = 2$$

$$\frac{a}{b+c}$$

\neq

$$\frac{a}{b} + \frac{a}{c}$$

$$\frac{2}{4+6} = \frac{2}{10} = \frac{1}{5}$$

$$3\left(\frac{2}{4}\right) + \left(\frac{2}{6}\right)^2$$

$$\frac{6}{12} + \frac{4}{12} = \frac{10}{12} = \frac{5}{6}$$

Section 1.3
Equivalent
Expressions

$$a=2$$

$$b=4$$

$$c=6$$

Examples

Using what we know determine if the following expressions are equivalent or not.

Section 1.3
Equivalent
Expressions

$$(9 + 6x) / 3$$

$$\begin{aligned} & (9 + 6(2)) / 3 \\ & (9 + 12) / 3 \\ & 21 / 3 = 7 \end{aligned}$$

$$2x^2$$

$$2^2 = 4 \cdot 2 = 8$$

$$3 + 6x$$

$$\begin{aligned} & 3 + 6(2) \\ & \frac{3 + 12}{15} \end{aligned}$$

$$(2x)^2$$

$$2 \cdot 2 = 4^2 = 16$$

Examples

Section 1.3 Equivalent Expressions

Example #2

Italian coffee costs 7 dollars per pound and Kenyan coffee costs 10 dollars per pound. Write an expression for the total amount spent on these coffees if you buy m pounds of Italian coffee and n pounds of Kenyan coffee.

$$7m + 10n$$

Write an expression for the sum of three consecutive integers, if the first integer is n .

$$1 + 2 + 3$$

$$n + (n+1) + (n+2)$$

Examples

Section 1.3 Equivalent Expressions

When we say that two expressions, such as $x + x$ and $2x$, are equivalent we are really saying: "For all numbers x , we have $x + x = 2x$." This statement looks like an equation.

In order to distinguish this use of equations, we refer to $x + x = 2x$ as an **identity**.

An **identity** is really a special equation, one that is satisfied by all values of the variables

Practice

Section 1.3 Equivalent Expressions

Find a value for x that will show the two expressions are not equivalent

$$2x + 8 \quad \text{and} \quad x + 4$$

$$\begin{aligned} 2(2) + 8 \\ 4 + 8 \\ 12 \end{aligned}$$

$$\begin{aligned} 2 + 4 \\ 6 \end{aligned}$$

Not an Identity

Are the expressions equivalent?

$$(x - y) + z \quad \text{and} \quad x - (y + z)$$

$$\begin{aligned} (2 - 4) + 6 \\ -2 + 6 \\ 4 \end{aligned}$$

$$\begin{aligned} 2 - (4 + 6) \\ 2 - 10 \\ -8 \end{aligned}$$

Not an identity

$$\begin{aligned} x &= 2 \\ y &= 4 \\ z &= 6 \end{aligned}$$

Practice

Section 1.3 Equivalent Expressions

Are the following equations identities?

$$3x + x = 4x$$

$$\begin{aligned} 3(2) + 2 &= 4(2) \\ 6 + 2 &= 8 \\ 8 &= 8 \checkmark \end{aligned}$$

$$3x + x = 4x$$

$$\begin{aligned} 3x + 1x &= 4x \\ 4x &= 4x \end{aligned}$$

$$2x^2 + 3x^4 = 5x^6$$

$$2(4)^2 + 3(4)^4 = 5(4)^6$$

$$2(16) + 3(256) = 5(4096)$$

$$32 + 768 = 20480$$

Key Points

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- When are expressions equivalent?
- Evaluating expressions to see when they are equal
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Section 1.4 Equivalent Equations

- When are equations equivalent?
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- Isolating variables
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Discussion

Section 1.4 Equivalent Equations

If you are given the equation $3x + 12 = 36$, how can we make it a simpler equation?

$$\begin{array}{r} 3x + 12 = 36 \\ -12 \quad -12 \\ \hline 3x = 24 \\ \frac{3}{3} \quad \frac{24}{3} \\ \hline x = 8 \end{array}$$

$$\begin{array}{r} 3x + 12 = 36 \\ \frac{3}{3} \\ \hline x + 4 = 12 \\ -4 \quad -4 \\ \hline x = 8 \end{array}$$

Discussion

Section 1.4 Equivalent Equations

What we have done is something called **isolating the variable**.

What does isolating the variable allow us to do?

When we isolate the variable we are finding the value that will make the equation true.

$$\underline{x=8}$$

$$3(8)+12=36$$

$$24+12=36$$

$$36=36$$

Examples

Section 1.4 Equivalent Equations

Without solving explain why each pair of equations have the same solution.

- a. $\cancel{4}(w-2)^2 = \cancel{4}6$ $4 \cdot (w-2)^2 = \frac{6}{\cancel{4}} \cdot \cancel{4}$
 $(w-2)^2 = \cancel{6}4$ $4(w-2)^2 = 6$
- b. $\cancel{12} \frac{x-4}{\cancel{12}} = -3 \cdot \cancel{12}$ $\frac{x-4}{12} = \frac{-36}{12}$
 $x-4 = -36$ $\frac{x-4}{12} = -3$
- c. $\begin{array}{r} y^4 + 3y + 4 = y^4 + 2 \\ -y^4 \quad \quad -y^4 \\ \hline 3y + 4 = 2 \end{array}$ $3y + 4 = 2$
 $y^4 + 3y + 4 = y^4 + 2$

Vocabulary

Section 1.4 Equivalent Equations

What we have just done is found a way to make two equations equivalent.

EQUIVALENT EQUATIONS

We say two equations are *equivalent* if they have exactly the same solutions

Examples

Section 1.4 Equivalent Equations

In the following equations we are going to isolate the variable, using reverse operations.

1. $5x - 4 = 26$

$$\begin{array}{r} +4 \quad +4 \\ \hline 5x = 30 \\ \hline 5 \quad 5 \end{array}$$

$$\boxed{x = 6}$$

$$\begin{array}{l} 5(6) - 4 = 26 \\ 30 - 4 = 26 \\ 26 = 26 \checkmark \end{array}$$

Examples

Section 1.4 Equivalent Equations

In the following equations we are going to isolate the variable, using reverse operations.

2. $\frac{1}{6}(8+x) = 10$

$$\cancel{6} \cdot \frac{1}{\cancel{6}}(8+x) = 10 \cdot 6$$

$$1(8+x) = 60$$

$$\begin{array}{r} 8+x=60 \\ -8 \quad -8 \\ \hline x=52 \end{array}$$

$$\cancel{\frac{6}{7}} \cdot \frac{1}{\cancel{6}}(8+x) = 10 \cdot \frac{6}{1}$$

$$8+x = \frac{60}{1}$$

$$\begin{array}{r} 8+x=60 \\ -8 \quad -8 \end{array}$$

$$x=52$$

Examples

Section 1.4 Equivalent Equations

In the following equations we are going to isolate the variable, using reverse operations.

3. $\frac{x-3}{7} = 1$

$$7 \cdot \frac{x-3}{7} = 1 \cdot 7$$

$$\begin{array}{rcl} x-3 & = & 7 \\ +3 & & +3 \end{array}$$

$$x = 10$$

General Info

Section 1.4 Equivalent Equations

We can transform an equation into an equivalent equation using any operation that does not change the balance between the two sides.

- Adding or subtracting the same number to both sides
- Multiplying or dividing both sides by the same number, provided that it is not zero
- Replacing any expression in an equation by an equivalent expression

$$3x+6$$
$$3(x+2)$$

Homework

Section 1.3
Equivalent
Expressions

Pages 16 - 17
2 - 12 even, 20 - 23, 34, 37

Section 1.4
Equivalent
Equations

Pages 23 - 24
2 - 16 even, 17 - 27 odd, 35 - 44