

KEY POINTS

Section 2.3 Expanding & Factoring

- Expanding using the distributive law
- Factoring quadratic expressions
- Perfect Squares
- Difference of squares

Review

Section 2.3 Expanding & Factoring

Simplify the following equations.

$$\begin{aligned} 1. \quad & \underline{2x^4} - \underline{3x} + 5 + \underline{4x^4} + 6x^2 - \underline{5x} \\ & 2x^4 + 4x^4 - 3x - 5x + 5 + 6x^2 \\ & 6x^4 - 8x + 5 + 6x^2 \\ & 6x^4 + 6x^2 - 8x + 5 \end{aligned}$$

$$\begin{aligned} 2. \quad & (3x^2 + 4x - 3) - (5x^2 + 9x - 4) \\ & 3x^2 + 4x - 3 - 5x^2 - 9x + 4 \\ & 3x^2 - 5x^2 + 4x - 9x - 3 + 4 \\ & -2x^2 - 5x + 1 \end{aligned}$$

Background

Section 2.3 Expanding & Factoring

There are many situations where we will expand mathematical situations and then combine like terms.

Given the following equation how would you simplify it?

$$(z - 4)(z - 2)$$

Background

Section 2.3

Expanding &
Factoring

One way to simplify it is by using the distributive law.

$$(z - 4)(z - 2)$$

$$z(z-2) - 4(z-2)$$

$$z^2 - 2z - 4z + 8$$

$$z^2 - 6z + 8$$

Try It!

Section 2.3
Expanding &
Factoring

$$\begin{aligned}& (3x - 2)(2x + 3) \\& 3x(2x + 3) - 2(2x + 3) \\& 6x^2 + 9x - 4x - 6 \\& 6x^2 + 5x - 6\end{aligned}$$

Try It!

Section 2.3
Expanding &
Factoring

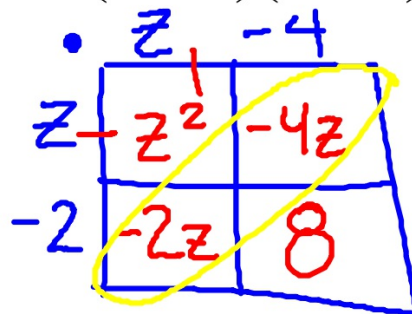
$$\begin{aligned}(w-9)^2 \\(w-9)(w-9) \\w(w-9)-9(w-9) \\w^2-9w-9w+81 \\w^2-18w+81\end{aligned}$$

Background

Section 2.3 Expanding & Factoring

Using the distributive law is one way to simplify our mathematical situations, however there are other ways to simplify that may make more sense to you. Let's do the rectangle diagram.

$$(z - 4)(z - 2)$$



$$z^2 - 6z + 8$$

Try It!

Section 2.3
Expanding &
Factoring

$$(3x - 2)(2x + 3)$$

	$3x$	-2
$2x$	$6x^2$	$-4x$
$+3$	$9x$	-6

$$6x^2 + 5x - 6$$

Background

We also have a method known as the foil method.

Section 2.3
Expanding &
Factoring

First
Outside
Inside
Last

$$\begin{array}{cccc} F & I & F & O \\ (z - 4) & (z - 2) \\ O & L & I & L \end{array}$$

$$\begin{array}{cccc} F & O & I & L \\ z \cdot z & + -2 \cdot z & + -4 \cdot z & + -4 \cdot -2 \\ z^2 & -2z & -4z & + 8 \\ z^2 & -6z & + 8 \end{array}$$

Try it!

Section 2.3
Expanding &
Factoring

$$\begin{array}{cccc} & & (w-9)^2 & \\ & \text{F} & \text{I} & \text{F} & \text{O} \\ (w-9) & \times & (w-9) & \\ & \text{O} & \text{L} & \text{I} & \text{L} \end{array}$$

$$w \cdot w - 9w - 9 \cdot w - 9 \cdot 9$$

$$w^2 - 9w - 9w + 81$$

$$w^2 - 18w + 81$$

Try it!

Section 2.3
Expanding &
Factoring

Pick one of the methods we used to combine like terms and simplify.

$$(x + y + z)(x - y - z)$$

	x	y	z
x	x^2	xy	xz
- y	$-xy$	$-y^2$	$-yz$
- z	$-xz$	$-yz$	$-z^2$

$$x^2 - y^2 - 2yz - z^2$$

Break Time!

Section 2.3
Expanding &
Factoring

Background

Section 2.3

Expanding & Factoring

x -9
 x x^2
 $+3$ -27

x^2	
	-27

In the first part of our lesson we were working with equations that were in factored form and changed them to general form. Now we are going to work backwards.

We are given an equation: $x^2 - 6x - 27$ (x)

9 -3
 -9 3
 1 -27
 -1 27
 $(x-9)(x+3)$

Identify coefficient with $x^2 = 1$
 multiply coeff. by our constant
 $1(-27) = -27$

-9 3
 9 -3 $x^2 - 9x + 3x - 27$
 -1 27 $x^2 - 6x - 27$
 1 -27 $(x-9)(x+3)$

Try it!

We are given an equation: $y^2 - 13y + 36$

Factors of 36

6	6
4	9
1	36
2	18

$$(y - 4)(y - 9)$$

Section 2.3
Expanding &
Factoring

Try it!

Section 2.3 Expanding & Factoring

We are given an equation: $\underline{2}x^2 + x - \underline{6}$

Identify coefficient: $\underline{2}$

Multiply by our constant: $\underline{2}(\underline{-6}) = -12$

Factors of -12 :

$1 - 12$

$-1 \ 12$

$2 - 6$

$-2 \ 6$

$3 - 4$

$-3 \ 4$

$$(2x^2 + 4x)(-3x - 6)$$

$$2x(x+2) - 3(x+2)$$

$$(2x-3)(x+2)$$

$$\underline{8}x^2 + 14x - \underline{15}$$

Coeff: 8

$$-15(8) = -120$$

Factors of -120

$$2 \quad 60$$

$$3 \quad 40$$

$$4 \quad 30$$

$$5 \quad 24$$

$$-6 \quad 20$$

$$10 \quad 12$$

$$8x^2 - 6x + 20x - 15$$

$$2x(4x - 3) + 5(4x - 3)$$

$$(2x + 5)(4x - 3)$$

Background

There are also so special rules that we can use, that don't require a lot of manipulation.

Section 2.3 Expanding & Factoring

Perfect Squares

$$(x + r)^2 = x^2 + 2xr + r^2$$

$$(x+4)^2 = x^2 + 2 \cdot x \cdot 4 + 4^2 \\ = x^2 + 8x + 16$$

$$(x+6)^2 = x^2 + 12x + 36$$

$$(x - r)^2 = x^2 - 2xr + r^2$$

$$(x-4)^2 = x^2 - 8x + 16$$

Difference of Squares

$$(x^2 - r^2) = (x - r)(x + r)$$

$$x^2 - 16 = (x+4)(x-4)$$

$$x^2 - 144 = (x+12)(x-12)$$

$$x^2 - 169 = (x+13)(x-13)$$

$$x^2 + 169 = (x+13i)(x-13i)$$

Try it!

Section 2.3
Expanding &
Factoring

$$z^2 - 225$$

$$(z + 15)(z - 15)$$

$$(x + 14)^2$$

$$x^2 + 28x + 196$$

$$(x - 14)^2$$

$$x^2 - 28x + 196$$

Homework

Section 2.3
Expanding &
Factoring

Pages 45-46
#2-24 even, 27-57 odd

Key Points

Section 2.4 Algebraic Fractions

- Canceling is really dividing a common factor from the numerator and denominator
- Canceling expressions is valid only when the factor being canceled is not zero
- Rules for operations on algebraic Fractions

Discussion

Section 2.4
Algebraic
Fractions

What are some things that you know about fractions?

Dividing fractions:
multiply the 1st fraction by the reciprocal of the 2nd

can be reduced

numerator

denominator

Part

Whole

exponents and variables

multiply fractions straight

add/subtract find a common denominator

Examples

Section 2.4 Algebraic Fractions

When we are dealing with algebraic fractions, the easiest way to approach them is to first see if anything can factor out, if something factors out it will make the fraction easier to work with.

$$\frac{10x+10}{5}$$
$$\frac{\cancel{5}(2x+2)}{\cancel{5}} = 2x+2 = 2(x+1)$$

$$\frac{10(x+1)}{5} = \frac{10}{5} \cdot \frac{x+1}{1} = 2(x+1)$$

Examples

Section 2.4
Algebraic
Fractions

$$\frac{4m-8n}{-6m-3n}$$

$$\frac{4m-8n}{-6m-3n} = \frac{4(m-2n)}{-3(2m+n)}$$

Examples

Section 2.4
Algebraic
Fractions

$$\frac{6+x}{2x+12}$$

$$\frac{6+x}{2x+12} = \frac{6+x}{2(x+6)} = \frac{\cancel{x+6}}{2\cancel{(x+6)}} = \frac{1}{2}$$

$$\frac{6+0}{2(0)+12} = \frac{6}{12} = \frac{1}{2}$$

Examples

Section 2.4
Algebraic
Fractions

$$\frac{2}{x+y} \cdot \frac{x^2y}{5}$$

$$\frac{2 \cdot x^2y}{5(x+y)} = \frac{2x^2y}{5(x+y)}$$

Examples

Section 2.4
Algebraic
Fractions

$$\frac{t^2-4t+3}{t^2-4} \cdot \frac{t-2}{t-3}$$

$$\frac{\begin{array}{r} t^2-4t+3 \\ 3 \\ -1, 3 \\ (t-1)(t-3) \end{array}}{\begin{array}{r} t^2-4 \\ (t-2)(t+2) \end{array}}$$

$$\frac{(t-1)\cancel{(t-3)}}{\cancel{(t-2)}(t+2)} \cdot \frac{\cancel{t-2}}{\cancel{t-3}}$$

$$\frac{(t-1)}{(t+2)}$$

$$\frac{z^2 + z - 6}{z^2 - 1} \cdot \frac{z-1}{z-2}$$

$$\frac{(z+3)(\cancel{z-2})}{(\cancel{z-1})(z+1)} \cdot \frac{\cancel{z-1}}{\cancel{z-2}} = \frac{z+3}{z+1}$$

Examples

Section 2.4
Algebraic
Fractions

$$\begin{aligned}
 & \frac{(x^2 - x - 12)/4x}{(x^2 + 2x - 3)/4x - 4} \\
 & \frac{x^2 - x - 12}{4x} \div \frac{x^2 + 2x - 3}{4x - 4} \\
 & \frac{x^2 - x - 12}{4x} \cdot \frac{4x - 4}{x^2 + 2x - 3} \\
 & \frac{(x-4)\cancel{(x+3)}}{\cancel{4}x} \cdot \frac{\cancel{4}\cancel{(x-1)}}{\cancel{(x-1)}\cancel{(x+3)}} = \frac{x-4}{x}
 \end{aligned}$$

Warm-Up

$$\frac{(r^2 - 25)/5r}{(r^2 - 10r + 25)/5r - 25}$$

$$\frac{r^2 - 25}{5r} \div \frac{r^2 - 10r + 25}{5r - 25}$$

$$\frac{r^2 - 25}{5r} \cdot \frac{5r - 25}{r^2 - 10r + 25}$$

$$\frac{(r+5)\cancel{(r-5)}}{\cancel{5}r} \cdot \frac{\cancel{5}\cancel{(r-5)}}{(\cancel{r-5})(\cancel{r-5})} = \frac{r+5}{r}$$

$$23. \frac{5v^2+15v}{v^2-v} \cdot \frac{3w+3}{5v+15}$$

$$\frac{\cancel{5}v(\cancel{v+3})}{v(v-1)} \cdot \frac{3(w+1)}{\cancel{5}(\cancel{v+3})} = \frac{3(w+1)}{w-1}$$

$$14. \frac{3xy^2}{4x^2z} \cdot \frac{8xy^3z}{6xy^5} = \frac{\cancel{2}4x^2\cancel{y}^5\cancel{z}}{\cancel{2}4x^3\cancel{y}^5\cancel{z}} = \frac{x^2}{x^3} = \frac{\cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot x} = \frac{1}{x}$$

Practice

Section 2.4
Algebraic
Fractions

Simplify

$$\frac{6x-6}{3}$$

$$\frac{\cancel{3}(2x-2)}{\cancel{3}} \left(\frac{2x-2}{1} \right)$$

$$\frac{\cancel{2}(x-1)}{\cancel{2}} = 2(x-1)$$

Practice

Section 2.4
Algebraic
Fractions

Simplify

$$\frac{10m+20n}{-15m-5n}$$

$$\frac{-2 \cancel{10}(m+2n)}{-5 \cancel{3}(3m+n)}$$

$$\frac{-2(m+2n)}{3m+n}$$

Practice

Section 2.4
Algebraic
Fractions

Simplify

$$\frac{5a^3+10a}{10a^2+20}$$

$$\frac{5a(a^2+2)}{10(a^2+2)}$$

$$\frac{5a}{2\cancel{10}} = \frac{a}{2} \text{ or } \frac{1a}{2}$$

Practice

Section 2.4
Algebraic
Fractions

Factor and simplify $\frac{3-x}{2x-6}$

$$\begin{aligned}\frac{3-x}{2x-6} &= \frac{3-x}{2(x-3)} = \frac{-x+3}{2(x-3)} \\ &= \frac{-1(\cancel{x-3})}{2(\cancel{x-3})} \\ &= \left(-\frac{1}{2}\right)\end{aligned}$$

Practice

Section 2.4
Algebraic
Fractions

Factor and simplify $\frac{2k^2-8}{2-k}$

$$\frac{2(k^2-4)}{2-k} = \frac{2(k-2)(k+2)}{2-k}$$

$$\frac{2(k-2)(k+2)}{-k+2} = \frac{2(k-2)(k+2)}{-1(k-2)}$$

$$\frac{2(k+2)}{-1} = -2(k+2)$$

Practice

Section 2.4
Algebraic
Fractions

Express as a single fraction

$$\frac{j}{7} + \frac{2j}{7}$$

$$\frac{\cancel{1}j}{\cancel{7}} + \frac{2j}{\cancel{7}} = \frac{\cancel{1}j + 2j}{\cancel{7}} = \frac{3j}{7}$$

Practice

Section 2.4
Algebraic
Fractions

Express as a sum of two algebraic fractions

$$\frac{5x+y}{6}$$

$$\frac{5x}{6} + \frac{y}{6}$$

Practice

Section 2.4
Algebraic
Fractions

Simplify the expression

$$\frac{30}{30} \left(\frac{c}{25} \right) + \left(\frac{h}{30} \right) \frac{25}{25}$$

$$\frac{30c}{750} + \frac{25h}{750} = \frac{30c + 25h}{750}$$

$$\frac{\cancel{5}(6c + 5h)}{\underset{150}{\cancel{750}}} = \frac{6c + 5h}{150}$$

Practice

Express as a single fraction.

Section 2.4
Algebraic
Fractions

pg 53-54

13-24

1-11 odd

26-42 even

51-55

$$\frac{p+2}{p+2} \left(\frac{3}{p} \right) + \left(\frac{2}{p+2} \right) \frac{p}{p}$$

$$\frac{3(p+2)}{p(p+2)} + \frac{2p}{p(p+2)}$$

$$\frac{3p+6}{p^2+2p} + \frac{2p}{p^2+2p} = \frac{5p+6}{p^2+2p} = \frac{5p+6}{p(p+2)}$$

Practice

Section 2.4
Algebraic
Fractions

Express as a single fraction

$$\frac{t}{s+t} - \frac{s}{s-t}$$

Practice

Section 2.4 Algebraic Fractions

Express as a single fraction

$$\frac{3}{v+w} * \frac{vw^2}{7}$$

$$\frac{3/(v+w)}{vw^2/7}$$

Practice

Section 2.4
Algebraic
Fractions

Express as a single fraction

$$\frac{z^2+z-6}{z^2-1} * \frac{z-1}{z-2}$$

Practice

Section 2.4
Algebraic
Fractions

Express as a single fraction

$$\frac{(r^2 - 25)/5r}{(r^2 - 10r + 25)/(5r - 25)}$$

Homework

Section 2.4
Algebraic
Fractions

Pages 53-54
1-11 odd, 14-24 even, 25-35, 51-55

