

KEY POINTS

Section 3.3

Absolute Value Equations and Inequalities

- Geometric definition of absolute value
- Algebraic definition of absolute value
- Absolute value equations
- Absolute value inequalities

Warm - Up

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Inequalities

$$-5(x + 9) - 8 \geq 32$$

$$-5x - 45 - 8 \geq 32$$

$$-5x - \cancel{53} \geq 32$$
$$+ \cancel{53} \quad + 53$$

$$\underline{-5x \geq 85}$$

$$\underline{-5} \quad \underline{-5}$$

$$x \leq -17$$

Discussion

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What do you know about absolute value?

makes all solutions positive

a negative on the outside of the absolute
value makes the solution negative

Discussion

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The difference between two numbers, say 5 and 3, depends on the order in which we subtract them.

Subtracting the smaller from the larger gives a positive number, $5 - 3 = 2$.

Subtracting the larger from the smaller gives a negative number, $3 - 5 = -2$.

Sometimes we are more interested in the *distance* between the two numbers than the difference.

Absolute value measures the distance of a number without distinguishing whether it is positive or negative.

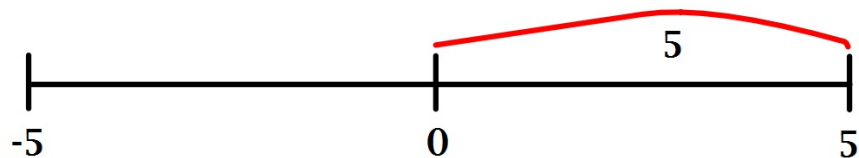
Examples

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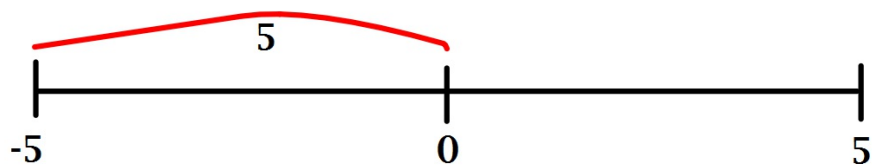
Absolute Value Equations and Inequalities

On the number line the *absolute value* of a number x , written as $|x|$, is the distance between 0 and x .

For example, $|5|$ is the distance between 0 and +5, so $|5|$ equals 5.



Likewise, $|-5|$ is the distance between 0 and -5, so $|-5|$ also equals 5.



Examples

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Evaluate the following expressions.

a.) $|100|$

$$= 100$$

b.) $|-3| = 3$

c.) $|0| = 0$

Examples

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Evaluate the following expressions.

a.) $|8 - 7| = |1| = 1$

b.) $|7 - 8| = |-1| = 1$

c.) $|7 + 8| = |15| = 15$

Examples

Section 3.3 Absolute Value Equations and Inequalities

Evaluate the following expressions.

a.) $|-8-9| = |-17| = 17$

$$-|-8-3| = -|-11|$$

b.) $|-2+9| = |7| = 7$

$$= -(11) \\ = -11$$

c.) $|-9-5| = |-14| = 14$

Examples

Section 3.3 Absolute Value Equations and Inequalities

$$|-3| = 3$$

$$|3| = 3$$

Solve for x. To solve for x when we are working with absolute value problems, we write two equations which allows us to solve for both possibilities of answers by setting our equation equal to both the positive and negative value of the given solution

$$|x - 5| = 4$$

$$\begin{array}{r} x - 5 = 4 \\ +5 \quad +5 \\ \hline \boxed{x = 9} \end{array}$$

$$|9 - 5| = 4$$

$$|4| = 4$$

$$4 = 4 \checkmark$$

$$\begin{array}{r} x - 5 = -4 \\ +5 \quad +5 \\ \hline \boxed{x = 1} \end{array}$$

$$|1 - 5| = 4$$

$$|-4| = 4$$

$$4 = 4 \checkmark$$

Practice

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$$|x + 1| = 5$$

$$\begin{array}{r} x + 1 = -5 \\ -1 \quad -1 \\ \hline x = -6 \end{array}$$

$$|-6 + 1| = 5$$

$$|-5| = 5$$
$$5 = 5 \checkmark$$

$$\begin{array}{r} x + 1 = 5 \\ -1 \quad -1 \\ \hline x = 4 \end{array}$$

$$|4 + 1| = 5$$

$$|5| = 5$$
$$5 = 5 \checkmark$$

Practice

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$$|x - 27| = -1$$

No Solution because the
absolute value will not give
a negative solution

Practice

$$|x - 7| = 4$$

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Practice

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$$\cancel{6} \cdot \frac{|x+5|}{\cancel{6}} = 2 \cdot \cancel{6}$$

$$|x+5|=12$$

$$\begin{array}{r} x+5=12 \\ -5 \quad -5 \\ \hline \boxed{x=7} \end{array}$$

$$\begin{array}{r} x+5=-12 \\ -5 \quad -5 \\ \hline \boxed{x=-17} \end{array}$$

$$\begin{array}{l} \frac{|7+5|}{6} = 2 \quad \frac{12}{6} = 2 \\ \frac{|12|}{6} = 2 \quad 2 = 2 \checkmark \end{array}$$

$$\begin{array}{l} \frac{|-17+5|}{6} \rightarrow \frac{12}{6} = 2 \\ \frac{|-12|}{6} = 2 \quad 2 = 2 \checkmark \end{array}$$

Practice

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$$3|2x - 5| - 7 = -1$$

$$\begin{array}{r} +7 \quad +7 \\ \hline 3|2x-5| = \frac{6}{3} \end{array}$$

$$|2x-5|=2$$

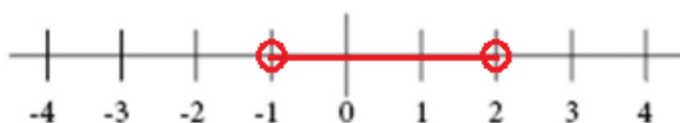
$$x = 7/2$$

$$x = 3/2$$

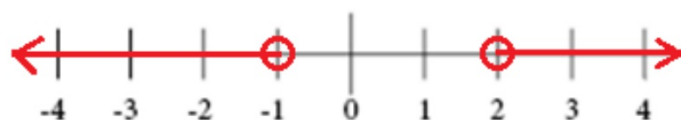
Examples

A compound inequality contains at least two inequalities that are separated by either "and" or "or".

The graph of a compound inequality with an "and" represents the intersection of the graph of the inequalities. A number is a solution to the compound inequality if the number is a solution to both inequalities. It can either be written as $x > -1$ and $x < 2$ or as $-1 < x < 2$.



The graph of a compound inequality with an "or" represents the union of the graphs of the inequalities. A number is a solution to the compound inequality if the number is a solution to at least one of the inequalities. It is written as $x < -1$ or $x > 2$.



Examples

Section 3.3 Absolute Value Equations and Inequalities

When we are working with inequalities that have absolute values we will use the same processes that we used when we solved absolute value equations. In order for $|2x - 3| < 7$ two things must be true:

$$2x - 3 > -7 \quad \text{and} \quad 2x - 3 < 7$$

In other words, $2x - 3$ must be between -7 and 7

$$2x - 3 > -7$$

$$2x > -4$$

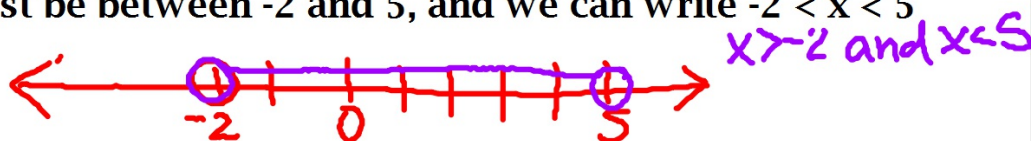
$$x > -2$$

$$2x - 3 < 7$$

$$2x < 10$$

$$x < 5$$

Since both these statements must be true, we see that x must be between -2 and 5 , and we can write $-2 < x < 5$



Examples

Section 3.3 Absolute Value Equations and Inequalities

When we are working with inequalities that have absolute values we will use the same processes that we used when we solved absolute value equations. In order for $|1 - 4x| \geq 10$ one of two things must be true:

$$1 - 4x \leq -10 \quad \text{or} \quad 1 - 4x \geq 10$$

In other words, $1 - 4x$ must not be between -10 and 10

$$1 - 4x \leq -10$$

$$-4x \leq -11$$

$$x \geq \frac{11}{4}$$

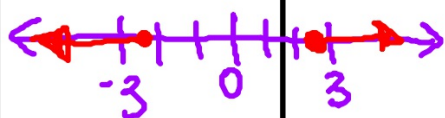
$$x \geq 2.75$$

$$1 - 4x \geq 10$$

$$-4x \geq 9$$

$$x \leq \frac{-9}{4}$$

$$x \leq -2.25$$



So, either $x \geq 2.75$ or $x \leq -2.25$

Examples

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$$|6 + 5w| < 8$$

$$\begin{array}{rcl}
 6 + 5w > -8 & & 6 + 5w < 8 \\
 -6 & -6 & -6 \\
 \hline
 5w > -14 & & 5w < 2 \\
 5 & 5 & 5 \\
 \hline
 w > -2.8 & & w < .4
 \end{array}$$

$\leftarrow \begin{array}{c} \bullet \quad \bullet \\ -2.8 \quad .4 \end{array} \rightarrow$

Practice

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$$|7 - 2x| > 4$$

$$\begin{array}{r} 7 - 2x > 4 \\ -7 \quad -7 \\ \hline -2x > -3 \\ \frac{-2}{-2} \quad \frac{-3}{-2} \\ x < 3\frac{1}{2} \\ x < 1.5 \end{array}$$

or

$$\begin{array}{r} 7 - 2x < -4 \\ -7 \quad -7 \\ \hline -2x < -11 \\ \frac{-2}{-2} \quad \frac{-11}{-2} \\ x > 11\frac{1}{2} \\ x > 5.5 \end{array}$$

Practice

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$$|2x - 3| < 7$$

Homework

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