

# KEY POINTS

## Section 4.4 Functions & Change

- Average rate of change
- Using units to interpret the average rate of change

## Section 4.5 Functions & Modeling

- Modeling
- Directly proportional

# Discussion

Section 4.4  
Functions &  
Change

What information can you remember about slope and finding slope?

$\frac{\text{rise}}{\text{run}}$

$$y = mx + b$$

↑  
slope

Rate of change

$(x_1, y_1) (x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

# Discussion

## Section 4.4 Functions & Change

When we are dealing with functions, they describe how quantities change. Functions compare values of the function for different inputs, which show how fast the output changes.

When we are working with functions we refer to the slope of the function as the **average rate of change**.

# Discussion

## Section 4.4 Functions & Change

When we look at slope, the slope formula is

$$\frac{\Delta y}{\Delta x} = \frac{\text{change in } y \text{ values}}{\text{change in } x \text{ values}} = \frac{y_2 - y_1}{x_2 - x_1}$$

When we look at average rate of change, the formula is

$$\frac{\Delta f(x)}{\Delta x} = \frac{\text{change in function values}}{\text{change in } x \text{ values}} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

# Discussion

## Section 4.4 Functions & Change

Another way of thinking about average rate of change is as follows:

Change in input = New  $x$ -value - Old  $x$ -value =  $b - a$

Change in output = New  $y$ -value - Old  $y$ -value =  $f(b) - f(a)$

$$\text{Average Rate of Change} = \frac{\text{Change in output}}{\text{Change in input}} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

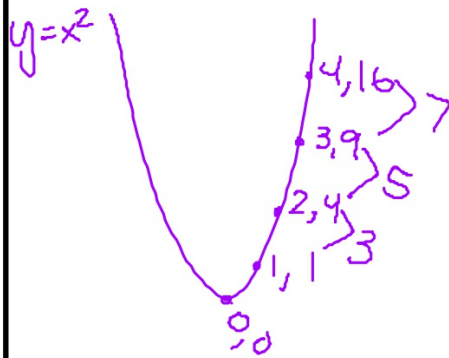
# Examples

## Section 4.4 Functions & Change

So what does it look like when working with functions

$$f(x) = x^2 \text{ between } x = 1^a \text{ and } x = 3^b$$

$$\frac{f(3) - f(1)}{3 - 1} = \frac{(3)^2 - (1)^2}{3 - 1} = \frac{9 - 1}{3 - 1} = \frac{8}{2} = 4$$



# Try It!

$$\frac{f(b) - f(a)}{b - a}$$

## Section 4.4 Functions & Change

$$f(x) = x^3 - 2x + 1 \text{ between } x = \overset{a}{-1} \text{ and } x = \overset{b}{4}$$

$$f(4) = 4^3 - 2(4) + 1$$

$$= 64 - 8 + 1$$

$$f(4) = 57$$

$$f(-1) = (-1)^3 - 2(-1) + 1$$

$$= -1 + 2 + 1$$

$$f(-1) = 2$$

$$\frac{57 - 2}{4 - (-1)} = \frac{55}{5}$$

$$= 11$$

$$f(x) = \frac{1}{x+1} \text{ between } x = \overset{a}{2} \text{ and } x = \overset{b}{5}$$

$$f(5) = \frac{1}{5+1} = \frac{1}{6}$$

$$\frac{\frac{1}{6} - (\frac{1}{3})^2}{5 - 2} = \frac{\frac{1}{6} - \frac{2}{6}}{3}$$

$$f(2) = \frac{1}{2+1} = \frac{1}{3}$$

$$\frac{-\frac{1}{6}}{3} = -\frac{1}{6} \div 3 = -\frac{1}{6} \cdot \frac{1}{3} = -\frac{1}{18}$$

# Example

## Section 4.4 Functions & Change

What about a word problem?

The total cost of production (in \$1000) is  $C(q) = q^3 - 12q^2 + 60q$ , where  $q$  is the number of thousands of units produced. Find the average rate of change between  $q = 1$  and  $q = 3$ . Interpret the results.

$$\text{Avg. ROC} = 25$$

$\frac{\text{cost of production}}{\text{\# of units produced}}$

25 dollars per each unit



# Examples

## Section 4.4 Functions & Change



A ball thrown in the air has a height  $h(t) = 90t - 16t^2$  feet after  $t$  seconds.

a.) What are the units of measurement for the average rate of change of  $h$ ? What does your answer tell you about how to interpret the rate of change in this case?

$$\frac{f(b) - f(a)}{b - a} = \frac{\text{feet}}{\text{sec}}$$

b.) Find the average rate of change of  $h$  between

i.)  $t = 0$  and  $t = 2$

ii.)  $t = 1$  and  $t = 2$

iii.)  $t = 2$  and  $t = 4$

$$122 \text{ ft/sec}$$

$$42 \text{ ft/sec}$$

$$-6 \text{ ft/sec}$$

# KEY POINTS

## Section 4.4 Functions & Change

- Average rate of change
- Using units to interpret the average rate of change

## Section 4.5 Functions & Modeling

- Modeling
- Directly proportional

# Discussion

## Section 4.5 Functions & Modeling

When we want to apply mathematics to a real-world situation, we are not always given a function: sometimes we have to find one.

Knowledge about the real world can help us to choose a particular type of function, and then we use the information about the exact situation we are modeling to select a function of this type.

# Discussion

## Section 4.5 Functions & Modeling

### DIRECT PROPORTIONALITY

Suppose a state sales tax rate is 6%. Then the tax,  $T$ , on a purchase of price  $P$  is given by the function

$$\text{Tax} = 6\% \times \text{Price} \quad \text{or} \quad T = 0.06P$$

Rewriting this equation as a ratio, we see that we are given a constant, in other words,

$$\frac{T}{P} = 0.06$$

This means that the tax is proportional to the purchase price.

# Discussion

## Section 4.5 Functions & Modeling

### DIRECT PROPORTIONALITY

A quantity  $Q$  is **directly proportional** to a quantity  $t$  if

$$Q = k \bullet t$$

where  $k$  is the *constant of proportionality*. We often leave out the word "directly" and simply say  $Q$  is proportional to  $t$ .

In the tax example, the constant of proportionality is  $k=0.06$ , because  $T = 0.06P$ .

# Examples

## Section 4.5 Functions & Modeling

State the constant of proportionality.

a.)  $y = 12n$

$$\frac{y}{n} = \frac{12n}{n}$$

$$\frac{y}{n} = 12 \Rightarrow \text{constant of proportionality } (k)$$

b.)  $\frac{C}{r} = \frac{2\pi r}{r}$

$$\frac{C}{r} = 2\pi = k$$

c.)  $y = x$

$$y = 1x$$

$$k = 1$$

d.)  $y = mx$

$$m = k \text{ (constant)}$$

e.)  $\frac{y}{x} = \frac{\sqrt{12}x}{x}$

$$\frac{y}{x} = \sqrt{12} = k$$

# Examples

## Section 4.5 Functions & Modeling

State whether the following equations describe direct proportions.

a.)  $Q = \frac{5}{t}$  Not directly proportional

$$t \cdot Q = \frac{5}{t} \cdot t$$

$$tQ = 5$$

b.)  $y = 3x$  directly proportional

$$\frac{y}{x} = 3$$

c.)  $y = 3x + 8$   $\frac{y-8}{x} = 3$

$$\frac{y-8}{x} = \frac{3x}{x}$$

Not directly proper.

d.)  $A = \frac{\pi r^2}{r^2}$

$$\frac{A}{r^2} = \pi$$

Not

e.)  $y = \frac{x^2}{x^2}$

$$\frac{y}{x^2} = 1$$

No

f.)  $Q = \frac{t}{5} \cdot 5$

$$\frac{5Q}{Q} = \frac{t}{Q}$$

$$5 = \frac{t}{Q}$$

Yes

# Examples

## Section 4.5 Functions & Modeling

The constant of proportionality is often thought of as a rate.

Identify the constant of proportionality in the example and give its units.

a.)  $D = 36t$ , where  $D$  is in miles and  $t$  is in hours

$$k = 36 \text{ miles/hour}$$

b.) I worked 8 hours and got paid \$96.

$$\frac{96}{8} = \$12/\text{hr}$$



# Homework

Section 4.4  
Functions &  
Change

Pages 99 - 100  
#1 - 10 all, 13-16

Section 4.5  
Functions &  
Modeling

Pages 106  
#1 - 13 all, 29

