

KEY POINTS

Section 5.2 Working with Linear Expressions

- Identifying linear expressions
- Different purposes of different forms
- Slope-intercept and point-slope form

Section 5.3 Solving Linear Equations

- Solving linear equations of the form $\text{linear expression} = \text{constant}$
- Solving linear equations of the form $\text{linear expression} = \text{linear expression}$
- Predicting properties of solutions
- Not all linear equations have exactly one solution

Warm-Up

Section 5.2
Working
with Linear
Expressions

What does a linear equation allow us to see?

Slope of a line
y intercepts
direction of the line

What are the different ways we can interpret a linear equation?

graphs
tables
word problems.

Discussion

Section 5.2
Working
with Linear
Expressions

$$b + mx$$

This is a linear expression in x ; that is x is the variable.

In order of this to be a linear expression the exponent of x must be a 1.

Remember from earlier lessons, an expression has no equal sign, which is why $b + mx$ is an expression.

Example

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Working
with Linear
Expressions

State the values of m and b in the following linear expressions.

a. $3x + 2$

$$m = 3$$

$$b = 2$$

c.) $3(5-2x)$

$$15 - 6x$$

$$m = -6 \quad b = 15$$

b.) $(x-1)/2$

$$\frac{x-1}{2} = \frac{x}{2} - \frac{1}{2} = \frac{1}{2}x - \frac{1}{2}$$

$$m = \frac{1}{2} \quad b = -\frac{1}{2}$$

d.) $0.5 - 0.2x$

$$m = -0.2$$

$$b = 0.5$$

Example

Section 5.2 Working with Linear Expressions

State whether the following expressions are linear.

a.) $3x - x^2 + 5$

Not Linear

c.) $\frac{x}{5} + \frac{5}{2}$

$\frac{1}{5}x + \frac{5}{2}$ Linear

e.) $3(3 + 4x)$

$9 + 12x$
Linear

b.) $\frac{5}{x} + \frac{x}{2}$

Not Linear

$5 \cdot \frac{1}{x} + \frac{1}{2}x$

$5x^{-1} + \frac{1}{2}x$

d.) $x(3 + 4x)$

$3x + 4x^2$

Not Linear

f.) $3 - 3^x$

Not Linear

Example

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Identify the constant term and the coefficient in the expression for the following linear functions.

a.) $u(t) = 20 + 4t$

Constant = 20
Coefficient = 4

b.) $v(t) = 8 - 0.3t$

Const = 8
Coeff = -0.3

c.) $w(t) = t/7 + 5$

Constant = 5
Coeff. = $1/7$

$t/7 = 1/7 t$

Warm-Up

Section 5.2 Working with Linear Expressions

Is the expression linear, justify your reasoning. If so, identify the constant and coefficient.

1. $(3z+4)/z$

$$\frac{3z+4}{z} = \frac{3z}{z} + \frac{4}{z}$$

Not linear because the denominator is a variable

2. $6^2 + (2/5)x$

Const $\rightarrow 36 + \frac{2}{5}x$ ← coeff.
Linear

Discussion

Section 5.2 Working with Linear Expressions

The form that we have been using for linear functions is called the **SLOPE-INTERCEPT FORM**.

The form

$$\begin{array}{l} f(x) = b + mx \quad \text{or} \quad y = b + mx \\ f(x) = mx + b \quad \text{or} \quad y = mx + b \end{array}$$

is used to express a linear function, because it shows the slope, m , and the vertical intercept, b , of the graph.

$b = \text{vertical intercept} = y\text{-intercept}$



Example

Section 5.2 Working with Linear Expressions

The cost C of a vacation lasting d days consists of the air fare, \$350, plus accommodation expenses of \$55 times the number of days, plus food expenses of \$40 times the number of days.

a.) Give an expression for C as a function of d that shows air fare, accommodation, and food expenses separately.

$$C(d) = 350 + 55d + 40d$$

b.) Express the function in slope-intercept form. What is the significance of the vertical intercept and the slope?

$$C(d) = 350 + 95d$$

350 is the initial cost of airfare

95 is the expenses paid each day

Discussion

Section 5.2 Working with Linear Expressions

Although slope-intercept form is the simplest form, sometimes another form shows us a different aspect of a function. Sometimes we are not given information that allows us to use the slope-intercept form.

POINT-SLOPE FORM

The form

$$y = y_1 + m(x - x_1) \quad y - y_1 = m(x - x_1)$$
$$f(x) = y_0 + m(x - x_0) \quad \text{or} \quad y = y_0 + m(x - x_0)$$

expresses a linear function using a point and slope value.

- The graph will pass through a point (x_0, y_0)
- The slope, or rate of change, is m

Examples

Section 5.2 Working with Linear Expressions

$$y = y_1 + m(x - x_1)$$

The population of a town t years after it is founded is given by $P(t) = 16,000 + 400(t - 5)$

$$m = 400 \quad (5, 16000)$$

a.) What is the practical interpretation of the constants 5 and 16,000 in the expression for P ?

At 5 years the population is 16,000.

b.) Express $P(t)$ in slope-intercept form and interpret the slope and y-intercept.

$$P(t) = 16000 + 400(t - 5)$$

$$P(t) = 16000 + 400t - 2000$$

$$P(t) = 14000 + 400t$$

The initial population is 14000 and it increases by 400 people each year

Examples

Section 5.2

Working with Linear Expressions

Find linear functions satisfying:

a.) $f(5) = -8$ and the rate of change is -3

$$y = y_0 + m(x - x_0)$$

$$y = -8 + -3(x - 5)$$

$$y = -8 - 3(x - 5)$$

m
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b.) $g(5) = 20$ and $g(8) = 32$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{32 - 20}{8 - 5} = \frac{12}{3} = 4$$

$$y = 20 + 4(x - 5) \quad y = 20 + 4x - 20 \quad y = 4x$$

$$y = 32 + 4(x - 8) \quad y = 32 + 4x - 32 \quad y = 4x$$

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- Different purposes of different forms
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Warm-Up

Section 5.3 Solving Linear Equations

Write a linear function for the following situation.

a.) $f(-4) = 9$ and the rate of change is 5

$$y = y_1 + m(x - x_1)$$

$$y = 9 + 5(x + 4)$$

$$\frac{y_2 - y_1}{x_2 - x_1}$$

b.) $g(3) = 10$ and $g(-6) = 8$

$$\frac{8 - 10}{-6 - 3} = \frac{-2}{-9} = \frac{2}{9}$$

$$y = 10 + \frac{2}{9}(x - 3)$$

$$y = 8 + \frac{2}{9}(x + 6)$$

Discussion

Section 5.3 Solving Linear Equations

Consider the equation: $2000a - 5000 = 3000$.

What is the goal of this problem?

To get a by itself, in order to find a value that makes the statement true

$$\begin{array}{r} 2000a - 5000 = 3000 \\ + 5000 \quad + 5000 \\ \hline 2000a = 8000 \\ \hline \frac{2000a}{2000} = \frac{8000}{2000} \\ a = 4 \end{array}$$

Example

Section 5.3 Solving Linear Equations

The population of a small island was 200,000 in 1990. It increased by 4000 people per year. This population growth can be given by $P(t) = 200,000 + 4000t$. When does the population reach 300,000?

$$\begin{aligned} 300,000 &= 200,000 + 4000t \\ -200,000 &\quad -200,000 \\ 100,000 &= 4000t \\ /4000 &\quad /4000 \\ 25 &= t \end{aligned}$$

REMEMBER!

Section 5.3
Solving
Linear
Equations

MOVE ALL TERMS INVOLVING THE VARIABLE TO ONE SIDE, ALL CONSTANTS TO THE OTHER!

THEN, DEPENDING ON THE COEFFICIENT OF THE VARIABLE, DIVIDE OR MULTIPLY TO SOLVE THE EQUATION.

Examples

Section 5.3 Solving Linear Equations

SOLVE!

a.) $5x + 3 = 2x - 9$

$$\begin{array}{r} -2x \quad -3 \quad -2x \quad -3 \\ \hline 3x = -12 \end{array} \quad \boxed{= -4}$$

b.) $p + 10 = 2p - 20$

$$\begin{array}{r} +20 \quad +20 \\ p + 30 = 2p - 20 \\ -p \quad -p \quad 30 = p \end{array}$$

c.) $50 + 0.05d = 20 + 2(30 + 0.01d)$

$$50 + 0.05d = 20 + 60 + 0.02d$$

$$\begin{array}{r} 50 + 0.05d = 80 + 0.02d \\ -0.02d \quad -0.02d \end{array}$$

$$\begin{array}{r} 50 + 0.03d = 80 \\ -50 \\ \hline 0.03d = 30 \\ \cdot 0.03 \quad \cdot 0.03 \end{array}$$

$$d = 1000$$

Examples

Section 5.3 Solving Linear Equations

$$0=10$$

Solve.

e.) $3x + 5 = 3(x + 5)$

$$\begin{array}{r} 3x+5=3x+15 \\ -3x \quad -3x \\ \hline 0+5=0+15 \\ 5 \neq 15 \end{array}$$

No Solution

f.) $3x+5 = 3(x + 1) + 2$

$$3x+5 = 3x+3+2$$

$$3x+5 = 3x+5$$

$$\begin{array}{r} -3x \quad -3x \\ \hline 0+5=0+5 \\ -5 \quad -5 \end{array}$$

$$0=0$$

Multiple
Solutions

Homework

Section 5.2
Working
with Linear
Expressions

Pages 126 - 127
1-28 all

Section 5.3
Solving
Linear
Equations

Pages 134
1-16 all, 29 - 35 all, 37-39 all

Warm - Up

Incandescent light bulbs are cheaper to buy, but more expensive to operate than fluorescent bulbs.

Incandescent $f(t) = 0.50 + 0.004t$

Flourescent $g(t) = 5.00 + 0.001t$

How many hours of operation gives the same cost with either choice?

