

KEY POINTS

Section 5.4 Equations for Lines in the Plane

- Using the form of a linear equation in two variables $y = mx + b$, $y = y_0 + m(x - x_0)$, and $Ax + By = C$ to analyze its graph
- Equations of horizontal and vertical lines
- Slopes of parallel and perpendicular lines

Background

Section 5.4 Equations for Lines in the Plane

Let's look at the equation $y = 1 + 2x$.

x	y	<i>Coordinates</i>
-2	-3	$(-2, -3)$
-1	-1	$(-1, -1)$
0	1	$(0, 1)$
1	3	$(1, 3)$
2	5	$(2, 5)$

Remember that the graph of an equation in two variables is the set of points in the plane that satisfy the equation. This means that there are an infinite number of solutions.

Background

Section 5.4 Equations for Lines in the Plane

Look at the following three equations, which ones are linear?

$$y = 6 - 2x$$

yes linear

$$y = b - mx$$

$$m = -2$$

$$b = 6$$

$$\begin{array}{r} y - 4 = -2(x - 1) \\ +4 \quad +4 \end{array}$$

$$y = 4 - 2(x - 1)$$

yes linear

point-slope form

$$m = -2$$

$$\text{point} = (1, 4)$$

$$y = 4 - 2x + 2$$

$$y = 6 - 2x$$

Standard Form

$$\begin{array}{r} 4x + 2y = 12 \\ -4x \quad -4x \end{array}$$

$$\frac{2y}{2} = \frac{12}{2} - \frac{4x}{2}$$

$$y = 6 - 2x$$

linear

Example

Section 5.4 Equations for Lines in the Plane

Given the equation, identify the slope and vertical intercept.

a.) $y - 5 = 8(x+1)$

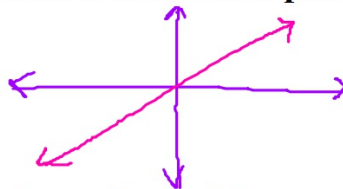
b.) $3x + 4y = 20$

c.) $6x - 15 = 2y - 3$

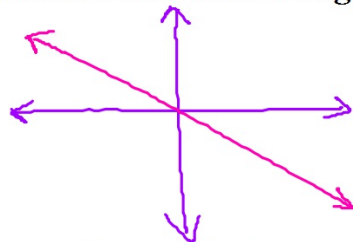
Discussion

Section 5.4 Equations for Lines in the Plane

We know that a line with a positive slope rises as we move from left to right.



We know that a line with a negative slope falls as we move from left to right.



What about a line with slope $m = 0$?

Examples

Section 5.4 Equations for Lines in the Plane

If you are given the equation $y = 4$, what is the slope and vertical intercept?

$$y = 4 + 0x$$

slope = 0
y int = 4

What would its graph look like?



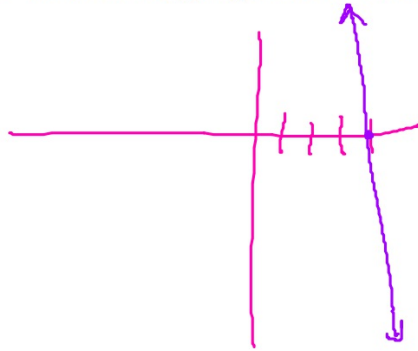
Examples

Section 5.4 Equations for Lines in the Plane

If you are given the equation $x = 4$, what is the slope and vertical intercept?

Slope = undefined
yint = None

What would its graph look like?



Discussion

Section 5.4 Equations for Lines in the Plane

The equation $x = 4$ cannot be put into slope-intercept form. This is because the slope is defined as:

A vertical line does not have an equation of the form $y = mx + b$ and its slope is undefined.

SO,

- The graph of the equation $y = k$ is a horizontal line and its slope is 0.
- The graph of the equation $x = k$ is a vertical line and its slope is undefined.

Discussion

Section 5.4 Equations for Lines in the Plane

The slope-intercept and point-slope forms for linear equations have one of the variables isolated on the left side of the equation. Linear equations can also be written in a form like $5x + 4y = 20$, where neither x or y is isolated on one side.

This is called *STANDARD FORM*.

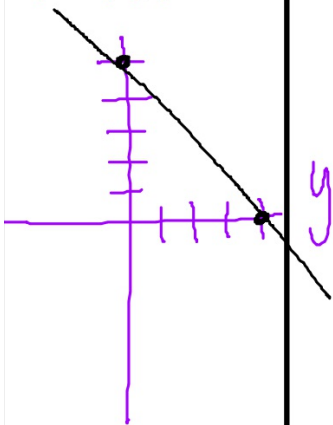
$Ax + By = C$; where A , B , and C are constants

$$y = y_0 + m(x - x_0)$$

$$y = mx + b$$

Example

Section 5.4 Equations for Lines in the Plane



Find the intercepts of the line $5x + 4y = 20$ and sketch a graph of the line.

$x = 0$ means we find the y-intercept

$$5(0) + 4y = 20$$

$$0 + \frac{4y}{4} = \frac{20}{4}$$

$$y = 5 \quad y_{\text{int}} = 5$$

$y = 0$ means we find the x-intercept

$$5x + 4(0) = 20$$

$$\frac{5x}{5} = \frac{20}{5}$$

$$x = 4 \quad x_{\text{int}} = 4$$

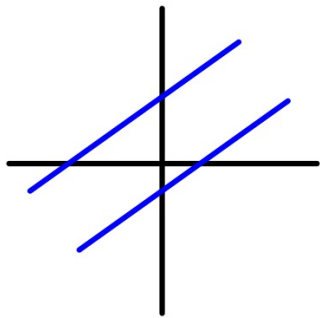
$$\text{Slope} = -\frac{5}{4}$$

$$\begin{array}{r} 5x + 4y = 20 \\ -5x \qquad \qquad -5x \\ \hline \frac{4y}{4} = \frac{20 - 5x}{4} \\ y = 5 - \frac{5}{4}x \end{array}$$

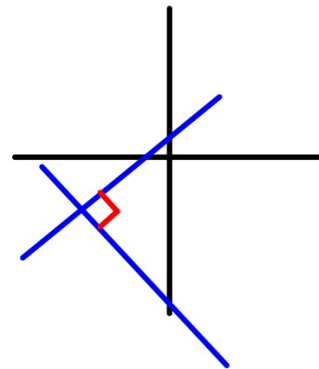
Discussion

Section 5.4 Equations for Lines in the Plane

The two samples below represent specific types of lines.



Parallel lines
They never intersect
They have the same
Slope



Perpendicular lines
Opposite slopes

Example

Section 5.4 Equations for Lines in the Plane

Show that the lines are parallel.

$$3x + 2y = 20$$

$$3x + 2y = 5$$

$$12x + 8y = 5$$

$$\begin{array}{r} 3x + 2y = 20 \\ - 3x \\ \hline 2y = 20 - 3x \\ \frac{2y}{2} = \frac{20 - 3x}{2} \\ y = 10 - \frac{3}{2}x \end{array}$$

$$\begin{array}{r} 3x + 2y = 5 \\ - 3x \\ \hline 2y = 5 - 3x \\ \frac{2y}{2} = \frac{5 - 3x}{2} \\ y = \frac{5}{2} - \frac{3}{2}x \end{array}$$

$$\begin{array}{r} 12x + 8y = 5 \\ - 12x \\ \hline 8y = 5 - 12x \\ \frac{8y}{8} = \frac{5 - 12x}{8} \\ y = \frac{5}{8} - \frac{12}{8}x \\ y = \frac{5}{8} - \frac{3}{2}x \end{array}$$

Example

Section 5.4 Equations for Lines in the Plane

Find an equation for

- a. The line parallel to the graph of $y = 12 - 3x$ with a y-intercept of 7.

$$y = 7 - 3x$$

- b. The line parallel to the graph of $5x + 3y = -6$ containing the point $(9, 4)$

$$\begin{array}{r} 5x + 3y = -6 \\ -5x \quad -5x \\ \hline 3y = -6 - 5x \\ \frac{3y}{3} = \frac{-6 - 5x}{3} \end{array}$$

$$y = -2 - \frac{5}{3}x$$

$$y = 4 - \frac{5}{3}(x - 9)$$

- c. The line perpendicular to the graph $y = 5x - 20$ that intersects the graph at $x = 6$

$$L = y = -\frac{1}{5}x - 20$$

$$y = 5(6) - 20$$

$$y = 30 - 20$$

$$y = 10$$

$$(6, 10)$$

$$y = 10 - \frac{1}{5}(x - 6)$$

Homework

Section 5.4
Equations
for Lines in
the Plane

Pages 141 - 143

~~#1-11~~ all, 12-20 even, 23-29 odd, 35-38, 51-55

1~4