

Do Now

Write a sentence explaining the pattern in each sequence. Then, find the next 2 terms.

1) 3, 7, 11...

2) 5, -3, -11...

3) 2.5, 5, 10...

4) 6, -3, 1.5...

5) 1, 4, 9, 16...

6) 1, 2, 6, 24...

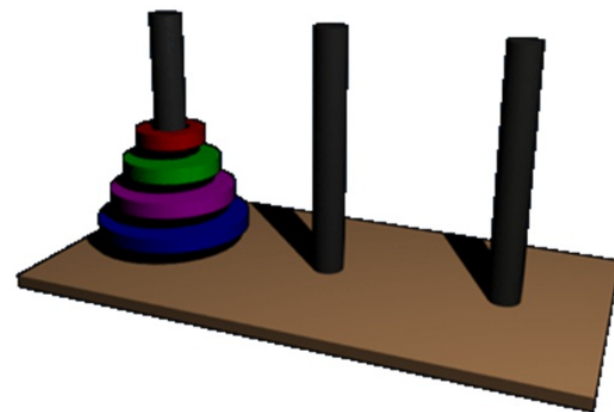
Learning Goal: Students will be able to determine if a sequence is arithmetic or geometric, and write a recursive formula to represent the sequence

Why?: Sequences can be used to model real-life scenarios

<https://www.mathsisfun.com/games/towerofhanoi.html>

Copy the TABLE:

# of Discs	Min # of Moves
2	3
3	7
4	15
5	31
6	?





Recursion is a process in which each step of a pattern is dependent on the step or steps that come before it.

A **sequence** is an ordered list of numbers.

Ex: 4, 6, 8, 10, 12, ...

Arithmetic Sequences

A sequence that increases by a constant amount each term

This constant is called the common difference

Example: 2, 5, 8, 11, 14, 17,.....

What is the common difference?

Geometric Sequences

A geometric sequence is a sequence in which each term is equal to the previous term multiplied by a constant.

This constant is called the common ratio

Example: 2, 6, 18, 54, 162,...

What is the common ratio?

Arithmetic vs Geometric

A concert hall has 59 seats in Row 1, 63 seats in Row 2, 67 seats in Row 3, and so on. The concert hall has 35 rows of seats. Write a recursive formula to find the number of seats in each row. How many seats are in Row 4? Which row has 95 seats?

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Take 5 minutes to look over the following problem from yesterday, be prepared to share out.

Arithmetic vs Geometric

A concert hall has 59 seats in Row 1, 63 seats in Row 2, 67 seats in Row 3, and so on. The concert hall has 35 rows of seats. Write a recursive formula to find the number of seats in each row. How many seats are in Row 4? Which row has 95 seats?

difference in seats

+ 4

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Recursive formulas are written to describe what to do to each term to get the next one.

Notation

u_n = the general nth (next term) term you are trying to define, *next term in sequence*

u_{n-1} = the previous term

Arithmetic

u_0 or u_1 = starting term

$$u_n = u_{n-1} + d$$

next term \uparrow u_n \nwarrow u_{n-1} *last term* $+$ d *common difference*

Geometric

u_0 or u_1 = starting term

$$u_n = r * u_{n-1}$$

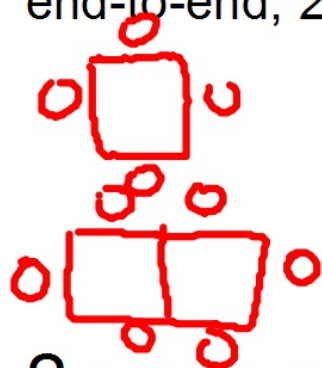
\uparrow *common ratio*

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Writing Recursive Formulas

1. A restaurant has 4-top tables (seats 4 customers). When the tables are placed end-to-end, 2 additional seats are added with each table added.



Tables	People
1	4
2	6

$$u_1 = 4$$

$$u_n = u_{n-1} + 2$$

$$u_2 = u_{2-1} + 2$$

$$u_2 = u_1 + 2$$

$$u_2 = 4 + 2$$

$$u_2 = 6$$

2. A population of honeybees is decreasing by 8% each year.

$$\text{Population} = 1000$$

$$u_n = 8\% \cdot 1000 - 1$$

$$y = 1000 \cdot .92^x$$

$$u_0 = 1000$$

$$u_n = .92 u_{n-1}$$

$$u_1 = .92(1000)$$

$$u_1 = 920$$

$$u_2 = .92(920)$$

$$u_2 = 846$$

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Why?: Sequences can be used to model real-life scenarios

CLASSWORK

Pages 34 - 36 # 5, 8, 11

Learning Goal: Students will be able to determine if a sequence is arithmetic or geometric, and write a recursive formula to represent the sequence

Why?: Sequences can be used to model real-life scenarios

5. Write a recursive formula to generate each sequence. Then find the indicated term.

a. 2, 6, 10, 14 Find the 15th term

b. 10, 5, 0, -5 Find the 12th term

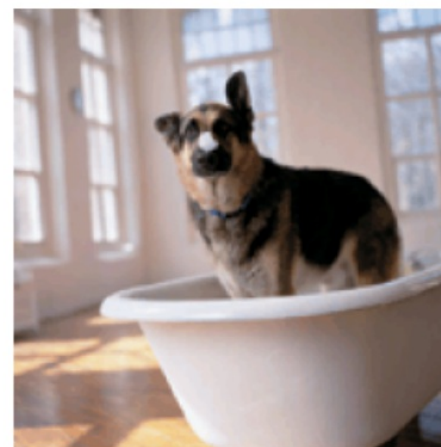
c. 0.4, 0.04, 0.004, 0.0004 Find the 10th term

d. -2, -8, -14, -20, -26 Find the 30th term

e. 1.56, 4.85, 8.14, 11.43 Find the 14th term

f. -6.24, -4.03, -1.82, 0.39 Find the 20th term

8. A 50-gallon (gal) bathtub contains 20 gal of water and is filling at a rate of 2.4 gal/min. You check the tub every minute on the minute.
- a. Suppose that the drain is closed. When will you discover that the water is flowing over the top?



- b. Now suppose that the bathtub contains 20 gal of water and is filling at a rate of 2.4 gal/min, but the drain is open and water drains at a rate of 3.1 gal/min. When will you discover that the tub is empty?
- c. Write a recursive formula that you can use to find the water level at any minute due to both the rate of filling and the rate of draining.

11. The week of February 14, the owner of Nickel's Appliances stocks hundreds of red, heart-shaped vacuum cleaners. The next week, he still has hundreds of red, heart-shaped vacuum cleaners. He tells the manager, "Discount the price 25 percent each week until they are gone."



- a.** On February 14, the vacuums are priced at \$80. What is the price of a vacuum during the second week?

- b.** What is the price during the fourth week?

- c.** When will the vacuum sell for less than \$10?

Lesson 1.1 • Recursively Defined Sequences

Name _____

Period _____

Date _____

1. Tell whether each sequence is arithmetic, geometric, or neither.

- | | | |
|------------------------|----------------------|---------------------------|
| a. 1, 5, 9, 13, ... | b. 2, 6, 18, 54, ... | c. 1, 1, 2, 3, 5, 8, ... |
| d. 16, 4, 1, 0.25, ... | e. -1, 1, -1, 1, ... | f. 5.6, 2.8, 0, -2.8, ... |

2. Find the common difference, d , for each arithmetic sequence and the common ratio, r , for each geometric sequence.

- | | | |
|-----------------------------|-------------------------------|--------------------------------|
| a. 6, 11, 16, 21, ... | b. 100, 10, 1, 0.1, ... | c. 1.5, 1.0, 0.5, 0, -0.5, ... |
| d. 0.0625, 0.125, 0.25, ... | e. -1, 0.2, -0.04, 0.008, ... | f. -4, -3.99, -3.98, ... |

3. Write the first six terms of each sequence, starting with u_1 .

- | | |
|---|--|
| a. $u_1 = -18$
$u_n = u_{n-1} + 6$ where $n \geq 2$ | b. $u_1 = 0.5$
$u_n = 3u_{n-1}$ where $n \geq 2$ |
| c. $u_1 = 35.6$
$u_n = u_{n-1} - 4.2$ where $n \geq 2$ | d. $u_1 = 8$
$u_n = -\frac{1}{2}u_{n-1}$ where $n \geq 2$ |

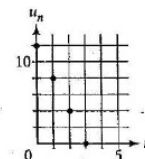
4. Write a recursive formula to generate each sequence. Then find the indicated term.

- | | |
|--|---------------------|
| a. -15, -11, -7, -3, ... | Find the 10th term. |
| b. 1000, 100, 10, 1, ... | Find the 12th term. |
| c. 17.25, 14.94, 12.63, 10.32, ... | Find the 15th term. |
| d. 0.3, -0.03, 0.003, -0.0003, ... | Find the 8th term. |
| e. $0, \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \dots$ | Find the 21st term. |
| f. -2, 4, -8, 16, ... | Find the 15th term. |

5. Indicate whether each situation could be represented by an arithmetic sequence or a geometric sequence. Give the value of the common difference, d , for each arithmetic sequence and of the common ratio, r , for each geometric sequence.

- Phil rented an apartment for \$850 a month. Each time he renewed his lease over the next 3 years, his landlord raised the rent by \$50.
- Leora was hired as a first-year teacher at an annual salary of \$30,000. She received an annual salary increase of 5% for each of the next 4 years.
- A laboratory technician observes that the number of bacteria in a colony doubles every 12 hours.
- The number of students enrolled in a high school is decreasing at a rate of 75 students per year.

6. Write a recursive formula for the sequence graphed at right. Find the 42nd term.



Lesson 1.2 • Modeling Growth and Decay

Name _____ Period _____ Date _____

1. Find the common ratio for each sequence.

- | | |
|-------------------------------------|-----------------------------------|
| a. 42, 126, 378, 1134, ... | b. 19.2, 3.84, 0.768, 0.1536, ... |
| c. 90, 99, 108.9, 119.79, ... | d. 1800, 1080, 648, 388.8, ... |
| e. 11.5, 43.7, 166.06, 631.028, ... | f. 375, 142.5, 54.15, 20.577, ... |

2. Identify each sequence in Exercise 1 as growth or decay and give the percent increase or decrease for each.

3. Write a recursive formula for each sequence in Exercise 1 and find the 6th term. Use u_1 for the first term given.

4. Factor each expression so that the variable appears only once.

- | | | | |
|-----------------------------|----------------------------|----------------------------|-------------------------------|
| a. $x + 0.25x$ | b. $y - 0.19y$ | c. $2A - 0.33A$ | d. $3B + 0.07B$ |
| e. $u_{n-1} + 0.085u_{n-1}$ | f. $u_{n-1} - 0.72u_{n-1}$ | g. $3u_{n-1} - 0.5u_{n-1}$ | h. $1.5u_{n-1} + 0.25u_{n-1}$ |

5. Find the percent increase or percent decrease in each situation. (Round to the nearest tenth of a percent.) Identify each change as an increase or a decrease.

- The number of students attending a high school grew from 1260 to 1580.
- A computer originally priced at \$2100 was put on sale for \$1850.
- When Melissa renewed her lease, her rent went up from \$780 to \$815.
- After laying off 560 employees, a company had 1266 employees left.
- The value of a car depreciated from \$15,900 to \$12,402.
- The population of a small town changed from 16,350 to 17,331.

6. Match each recursive formula to a graph.

a. $u_1 = 35$
 $u_n = (1 - 0.3) \cdot u_{n-1}$ where $n \geq 2$

b. $u_1 = 35$
 $u_n = (1 - 0.5) \cdot u_{n-1}$ where $n \geq 2$

c. $u_1 = 35$
 $u_n = -0.5 + u_{n-1}$ where $n \geq 2$

