

KEY POINTS

Section 2.3 Expanding & Factoring

- Expanding using the distributive law
- Factoring quadratic expressions
- Perfect Squares
- Difference of squares

Warm-Up

Section 2.3
Expanding &
Factoring

Simplify the following equations.

1. $\underline{2x^4} - 3x + 5 + \underline{4x^4} + 6x^2 - 5x$
 $2x^4 + 4x^4 - 3x - 5x + 5 + 6x^2$
 $6x^4 - 8x + 5 + 6x^2$
 $6x^4 + 6x^2 - 8x + 5$

2. $(3x^2 + 4x - 3) - (5x^2 + 9x - 4)$
 $3x^2 + 4x - 3 - 5x^2 - 9x + 4$
 $-2x^2 - 5x + 1$

Background

Section 2.3 Expanding & Factoring

There are many situations where we will expand mathematical situations and then combine like terms.

Given the following equation how would you simplify it?

$$(z - 4)(z - 2)$$

Substituting a value in
combine like terms
distributive law
FOIL

Background

Section 2.3 Expanding & Factoring

One way to simplify it is by using the distributive law.

$$(z - 4)(z - 2)$$

$$z(z - 2) - 4(z - 2)$$

$$z^2 - 2z - 4z + 8$$

$$z^2 - 6z + 8$$

Try It!

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$$(3x - 2)(2x + 3)$$

$$6x^2 + 9x - 4x - 6$$

$$6x^2 + 5x - 6$$

Try It!

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$$(w - 9)^2$$

$$(w - 9)(w - 9)$$

$$w^2 - 18w + 81$$

Background

Section 2.3 Expanding & Factoring

Using the distributive law is one way to simplify our mathematical situations, however there are other ways to simplify that may make more sense to you. Let's do the rectangle diagram.

$$(z - 4)(z - 2)$$

• $z^2 - 6z + 8$

Try It!

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$$(3x - 2)(2x + 3)$$

	$2x$	3
$3x$	$6x^2$	$9x$
-2	$-4x$	-6

$$6x^2 + 9x - 4x$$

$$6x^2 + 5x - 6$$

Background

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First
Outside
Inside
Last

We also have a method known as the foil method.

$$\begin{array}{cccc} F & L & F & O \\ (z - 4) & (z - 2) \\ O & I & I & L \end{array}$$

$$\begin{array}{cccc} F & O & I & L \\ z \cdot z & -2 \cdot z & -4 \cdot z & -4 \cdot -2 \\ z^2 & -2z & -4z & +8 \end{array}$$

$$\begin{array}{c} \vee \\ z^2 - 6z + 8 \end{array}$$

Try it!

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$$(w - 9)^2$$
$$(w - 9)(w - 9)$$

$$\begin{array}{cccc} \underline{F} & \underline{O} & \underline{I} & \underline{L} \\ w \cdot w & -9 \cdot w & -9 \cdot w & -9 \cdot -9 \end{array}$$

$$w^2 - 9w - 9w + 81$$

$$w^2 - 18w + 81$$

Try it!

Section 2.3 Expanding & Factoring

Pick one of the methods we used to combine like terms and simplify.

$$(x + y + z)(x - y - z)$$

	x	+y	+z
x	x^2	xy	xz
-y	$-xy$	$-y^2$	$-yz$
-z	$-xz$	$-yz$	$-z^2$

$$x^2 - y^2 - 2yz - z^2$$

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2-24 even

Warm-Up

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Simplify the following expression

1. $(2x + 3y)^2$

$$(2x + 3y)(2x + 3y)$$

$$4x^2 + 6xy + 6xy + 9y^2$$

$$4x^2 + 9y^2 + 12xy$$

$$24. 2x(5x+8)(7x+2)$$

	$5x$	$+8$
$7x$	$35x^2$	$56x$
$+2$	$10x$	$+16$

$$2x(35x^2 + 66x + 16) \quad (x-y)(x^2 - 2xy + y^2)$$

$$70x^3 + 132x^2 + 32x$$

	x^2	$-2xy$	$+y^2$
x	x^3	$-2x^2y$	xy^2
$-y$	$-x^2y$	$2xy^2$	$-y^3$

$$x^3 - 3x^2y + 3xy^2 - y^3$$

$$16. (x-y)^3$$

$$(x-y)(x-y)(x-y)$$

	x	$-y$
x	x^2	$-xy$
$-y$	$-xy$	$+y^2$

Background

Section 2.3 Expanding & Factoring

FACTORING QUADRATIC EXPRESSIONS

If a quadratic expression is factorable, the following steps work:

- Factor out all common constant factors, giving $k(ax^2+bx+c)$
- In the remaining expression, multiply the coefficient of the x^2 term by the constant term ac .
- Find two numbers that multiply to ac and sum to b , the coefficient of the x term
- Break the middle term, bx , into two terms using the result of the previous step
- Factor the four terms by grouping

Background

Section 2.3

Expanding & Factoring

In the first part of our lesson we were working with equations that were in factored form and changed them to general form. Now we are going to work backwards.

If possible, factor into $(x + r)(x + s)$, where r and s are integers.

We are given an equation: $x^2 - 6x - 27$

1. Identify coefficient attached to x^2 : 1
2. Multiply the coeff. by our constant: 1(-27)
3. Find the factors of your result.
4. Find the combo that is the sum of the terms -6

-27
-9 3
9-3
-1 27
1-27

$$(x-9)(x+3)$$

Try it!

We are given an equation: $x^2 - 9x + 18$

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$$\begin{array}{r} 18 \\ \hline 1 \quad 18 \\ -1 \quad -18 \\ -2 \quad -9 \\ 2 \quad 9 \\ 3 \quad 6 \\ -3 \quad -6 \end{array}$$

$$(x-3)(x-6)$$

Try it!

We are given an equation: $y^2 - 13y + 36$

$$(y-4)(y-9)$$

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Try it!

Section 2.3 Expanding & Factoring

We are given an equation: $x^2 + 10xy + 24y^2$

$$\begin{array}{r} 24 \\ \hline 1 \ 24 \\ 2 \ 12 \\ 3 \ 8 \\ 4 \ 6 \end{array}$$

$$(x+4y)(x+6y)$$

Another Issue

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Expanding &
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WHAT IF THE COEFFICIENT OF x^2 IS NOT 1?

$2x^2 + 7x + 3$
Identify coefficient: 2
multiply by constant: $2 \cdot 3 = 6$

Factors of 6: 1, 6
2, 3

$$\begin{aligned} & (2x^2 + 6x) + (x + 3) \\ & 2x(\cancel{x+3}) + 1(\cancel{x+3}) \\ & (x+3)(2x+1) \end{aligned}$$

Try it!

Section 2.3 Expanding & Factoring

We are given an equation: $2x^2 + x - 6$

$$2(-6) = -12$$

$$\begin{array}{r} -12 \\ \hline \end{array}$$

$$\begin{array}{rr} -1 & 12 \\ \hline \end{array}$$

$$\begin{array}{rr} 1 & -12 \\ \hline \end{array}$$

$$\begin{array}{rr} -2 & 6 \\ \hline \end{array}$$

$$\begin{array}{rr} 2 & -6 \\ \hline \end{array}$$

$$\begin{array}{rr} -3 & 4 \\ \hline \end{array}$$

$$\begin{array}{rr} 3 & -4 \\ \hline \end{array}$$

$$(2x^2 + 4x) - (3x - 6)$$

$$2x(x+2) - 3(x+2)$$

$$(2x-3)(x+2)$$

Try it!

We are given an equation: $8x^2 + 14x - 15$

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Try it!

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Expanding &
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We are given an equation: $12x^2 - 44x + 24$

$$4(3x^2 - 11x + 6)$$

$$3(6) = 18$$

$$\begin{array}{r} 18 \\ \hline \end{array}$$

$$1 \quad 18$$

$$\textcircled{-2 \quad -9}$$

$$36$$

$$(3x^2 - 9x) - (2x + 6)$$

$$3x(x-3) - 2(x+3)$$

$$4(x-3)(3x-2)$$

$$(6x-4)(2x-6)$$

Background

There are also so special rules that we can use, that don't require a lot of manipulation.

Section 2.3 Expanding & Factoring

Perfect Squares

$$(x + r)^2 = x^2 + 2xr + r^2$$

$$(x+4)^2 = x^2 + 2 \cdot x \cdot 4 + 4^2 \\ = x^2 + 8x + 16$$

$$(x+6)^2 = x^2 + 12x + 36$$

$$(x - r)^2 = x^2 - 2xr + r^2$$

$$(x-4)^2 = x^2 - 8x + 16$$

Difference of Squares

$$(x^2 - r^2) = (x - r)(x + r)$$

$$(x^2 - 16) = (x - 4)(x + 4)$$

$$(x^2 - 144) = (x - 12)(x + 12)$$

Background

Section 2.3 Expanding & Factoring

Perfect Squares

An expression with three terms is a PERFECT SQUARE if:

- Two of the terms are squares, and
- The third term is twice the product of the expressions whose squares are the other terms

Difference of Squares

If an expression is in the form $x^2 - r^2$, it can be factored as:

$$x^2 - r^2 = (x - r)(x + r)$$

Try it!

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$$z^2 - 225 \quad \sqrt{225} = 15$$
$$(z - 15)(z + 15)$$

$$(x + 14)^2 = x^2 + 28x + 196$$

$$(x - 14)^2 = x^2 - 28x + 196$$

Try it!

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$$49y^2 + 25$$

~~$(7y - 5)(7y - 5)$~~ $49y^2 - 25$
 $(7y + 5)(7y - 5)$

Not factorable

$$x^2 - 100$$

$$(x + 10)(x - 10)$$

Try it!

Section 2.3 Expanding & Factoring

$$4r^2 + 10r + 25$$

$$4r^2 = 2r \cdot 2r$$

$$25 = 5 \cdot 5$$

$$2(2r)(5) = 20r$$

Not a perfect square

$$9p^2 + 60pq + 100q^2$$

$$9p^2 = 3p \cdot 3p$$

$$100q^2 = 10q \cdot 10q$$

$$2(3p)(10q) = 60pq$$

Not a perfect square

$$25y^2 - 30yz + 9z^2$$

$$25y^2 = 5y \cdot 5y$$

$$9z^2 = 3z \cdot 3z$$
$$= -3z \cdot -3z$$

$$2(5y)(-3z) = -30yz$$

$$(5y - 3z)^2$$

Homework

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#2-24 even, 27-57 odd

