

# KEY POINTS

Section 3.3

**Absolute  
Value  
Equations  
and  
Inequalities**

- Geometric definition of absolute value
- Algebraic definition of absolute value
- Absolute value equations
- Absolute value inequalities

# Warm - Up

Section 3.3  
Absolute  
Value  
Equations  
and  
Inequalities

$$-5(x + 9) - 8 \geq 32$$

$$\begin{array}{r} +8 \quad +8 \\ \hline -5x - 45 \geq 40 \\ +45 \quad +45 \\ \hline \end{array}$$

$$\begin{array}{r} -5x \geq 85 \\ \hline -5 \quad -5 \\ \hline \end{array}$$

$$x \leq -17$$

# Discussion

Section 3.3  
Absolute  
Value  
Equations  
and  
Inequalities

What do you know about absolute value?

Solution is always positive

When dealing with equations, the absolute value has to be isolated before solving.

# Discussion

Section 3.3  
Absolute  
Value  
Equations  
and  
Inequalities

The difference between two numbers, say 5 and 3, depends on the order in which we subtract them.

Subtracting the smaller from the larger gives a positive number,  $5 - 3 = 2$ .

Subtracting the larger from the smaller gives a negative number,  $3 - 5 = -2$ .

Sometimes we are more interested in the *distance* between the two numbers than the difference.

Absolute value measures the distance of a number without distinguishing whether it is positive or negative.

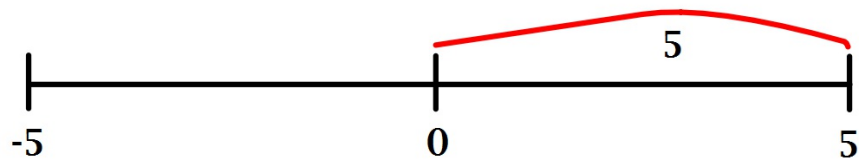
# Examples

## Section 3.3

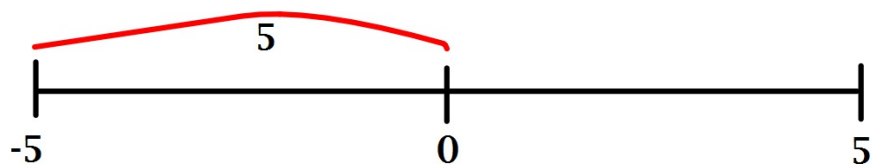
### Absolute Value Equations and Inequalities

On the number line the *absolute value* of a number  $x$ , written as  $|x|$ , is the distance between 0 and  $x$ .

For example,  $|5|$  is the distance between 0 and +5, so  $|5|$  equals 5.



Likewise,  $|-5|$  is the distance between 0 and -5, so  $|-5|$  also equals 5.



# Examples

Section 3.3  
Absolute  
Value  
Equations  
and  
Inequalities

Evaluate the following expressions.

a.)  $|100| = 100$

b.)  $|-3| = 3$

c.)  $|0| = 0$

# Examples

Section 3.3  
Absolute  
Value  
Equations  
and  
Inequalities

Evaluate the following expressions.

a.)  $|8 - 7| = |1| = 1$

b.)  $|7 - 8| = |-1| = 1$

$-1$

c.)  $|7 + 8| = |15| = 15$

# Examples

Section 3.3  
Absolute  
Value  
Equations  
and  
Inequalities

Evaluate the following expressions.

a.)  $|-8-9| = 17$

$$-|-8-3| = -|-11| \\ = -11$$

b.)  $|-2+9| = 7$

c.)  $|-9-5| = 14$

# Examples

## Section 3.3 Absolute Value Equations and Inequalities

Solve for x. To solve for x when we are working with absolute value problems, we write two equations which allows us to solve for both possibilities of answers by setting our equation equal to both the positive and negative value of the given solution

$$|x - 5| = 4$$

$$x - 5 = 4$$

$$\begin{array}{r} +5 \quad +5 \\ \hline x = 9 \end{array}$$

$$|9 - 5| = 4$$

$$|4| = 4$$

$$4 = 4 \checkmark$$

$$x - 5 = -4$$

$$\begin{array}{r} +5 \quad +5 \\ \hline x = 1 \end{array}$$

$$|1 - 5| = 4$$

$$|-4| = 4$$

$$4 = 4 \checkmark$$

# Practice

Section 3.3  
Absolute  
Value  
Equations  
and  
Inequalities

$$|x + 1| = 5$$

$$\begin{array}{ll} x + 1 = 5 & x + 1 = -5 \\ -1 & -1 \\ \hline x = 4 & x = -6 \\ 4 & 5 \end{array}$$

# Practice

Section 3.3  
Absolute  
Value  
Equations  
and  
Inequalities

$$|x - 27| = -1$$

No solution because the absolute  
value cannot have a negative  
solution

# Practice

Section 3.3  
Absolute  
Value  
Equations  
and  
Inequalities

$$|x - 7| = 4$$

$$\begin{array}{l|l} x - 7 = 4 & x - 7 = -4 \\ +7 & +7 \quad +7 \\ x = 11 & x = 3 \end{array}$$

# Practice

Section 3.3  
Absolute  
Value  
Equations  
and  
Inequalities

$$\cancel{6} \cdot \frac{|x+5|}{\cancel{6}} = 2 \cdot 6$$

$$|x+5| = 12$$

$$\begin{array}{r|l} x+5=12 & \\ -5 & -5 \\ \hline x=7 & \end{array}$$

$$\begin{array}{r|l} x+5=-12 & \\ -5 & -5 \\ \hline x=-17 & \end{array}$$

# Practice

Section 3.3  
Absolute  
Value  
Equations  
and  
Inequalities

$$\begin{array}{r} 3|2x - 5| - 7 = -1 \\ \quad \quad \quad +7 \quad +7 \\ \hline \cancel{3}|2x - 5| = \frac{6}{\cancel{3}} \\ |2x - 5| = 2 \end{array}$$

$$\begin{array}{l|l} 2x - 5 = 2 & 2x - 5 = -2 \\ x = 7/2 \text{ or } 3.5 & x = 3/2 \text{ or } 1.5 \end{array}$$

$$\frac{|3p+7|}{4} + 2 = 5$$

4      -2    -2

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pg 73-74  
1-25 all

~~$$4 \cdot \frac{|3p+7|}{4} = 3 \cdot 4$$~~

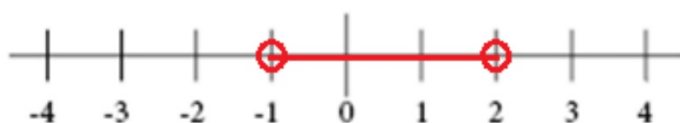
$$|3p+7| = 12$$

$$p = 5/3 \text{ and } -19/3$$

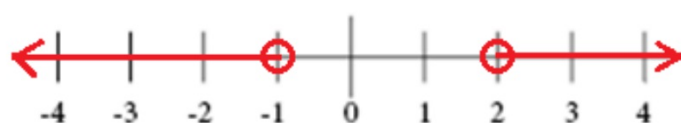
# Examples

A compound inequality contains at least two inequalities that are separated by either "and" or "or".

The graph of a compound inequality with an "and" represents the intersection of the graph of the inequalities. A number is a solution to the compound inequality if the number is a solution to both inequalities. It can either be written as  $x > -1$  and  $x < 2$  or as  $-1 < x < 2$ .



The graph of a compound inequality with an "or" represents the union of the graphs of the inequalities. A number is a solution to the compound inequality if the number is a solution to at least one of the inequalities. It is written as  $x < -1$  or  $x > 2$ .



# Examples

## Section 3.3 Absolute Value Equations and Inequalities

When we are working with inequalities that have absolute values we will use the same processes that we used when we solved absolute value equations. In order for  $|2x - 3| < 7$  two things must be true:

$$2x - 3 > -7 \quad \text{and} \quad 2x - 3 < 7$$

In other words,  $2x - 3$  must be between  $-7$  and  $7$

$$2x - 3 > -7$$

$$2x > -4$$

$$x > -2$$

$$2x - 3 < 7$$

$$2x < 10$$

$$x < 5$$

$$x > -2 \text{ and } x < 5$$

$$-2 < x < 5$$

$$x > 0$$

Since both these statements must be true, we see that  $x$  must be between  $-2$  and  $5$ , and we can write  $-2 < x < 5$



# Examples

## Section 3.3 Absolute Value Equations and Inequalities

When we are working with inequalities that have absolute values we will use the same processes that we used when we solved absolute value equations. In order for  $|1 - 4x| \geq 10$  one of two things must be true:

$$1 - 4x \leq -10 \quad \text{or} \quad 1 - 4x \geq 10$$

In other words,  $1 - 4x$  must not be between  $-10$  and  $10$

$$1 - 4x \leq -10$$

$$-4x \leq -11$$

$$x \geq \frac{11}{4}$$

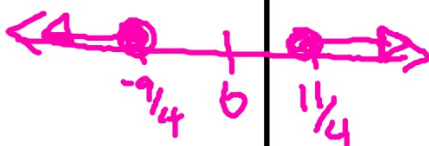
$$x \geq 2.75$$

$$1 - 4x \geq 10$$

$$\begin{array}{l} -4x \geq 9 \\ \underline{-4 \quad -4} \end{array}$$

$$x \leq \frac{-9}{4}$$

$$x \leq -2.25$$



So, either  $x \geq 2.75$  or  $x \leq -2.25$

# Examples

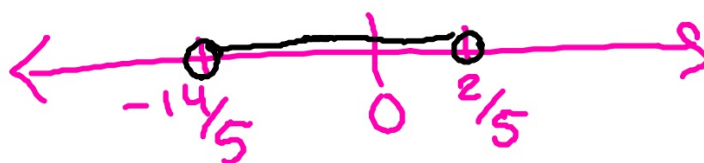
Section 3.3  
Absolute  
Value  
Equations  
and  
Inequalities

$$|6 + 5w| < 8$$

$$\begin{array}{r} 6 + 5w < 8 \\ -6 \quad -6 \\ \hline 5w < 2 \\ \frac{5w}{5} < \frac{2}{5} \\ w < \frac{2}{5} \end{array}$$

$$\begin{array}{r} 6 + 5w > -8 \\ -6 \quad -6 \\ \hline 5w > -14 \\ \frac{5w}{5} > \frac{-14}{5} \\ w > -\frac{14}{5} \end{array}$$

$$w > -\frac{14}{5} \text{ and } w < \frac{2}{5} \quad -\frac{14}{5} < w < \frac{2}{5}$$

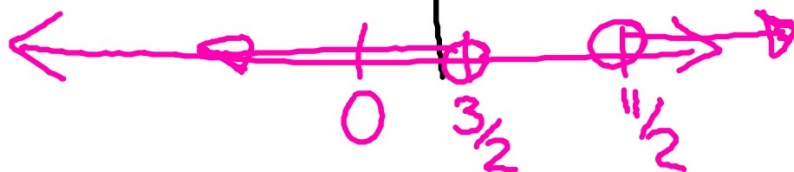


# Practice

Section 3.3  
Absolute  
Value  
Equations  
and  
Inequalities

$$|7 - 2x| > 4$$

$$\begin{array}{rcl} 7 - 2x > 4 & & 7 - 2x < -4 \\ -7 & & -7 \\ \hline -2x > -3 & & -2x < -11 \\ \frac{-2x}{-2} > \frac{-3}{-2} & & \frac{-2x}{-2} < \frac{-11}{-2} \\ x < 3/2 & & x > 11/2 \end{array}$$



# Practice

Section 3.3  
Absolute  
Value  
Equations  
and  
Inequalities

$$|2x - 3| < 7$$

# Homework

Section 3.3  
Absolute  
Value  
Equations  
and  
Inequalities

Pages 73-74  
#1-25 all

