

Warm-Up

Section 5.6
Systems of
Linear
Equations

Write down everything you know about systems of equations.

multiple equations

x and y value

substitution method

elimination method

graphing

one solution (x, y)

KEY POINTS

Section 5.6 Systems of Linear Equations

- Definition of a system of equations
- Solving linear systems using substitution
- Solving linear systems using elimination
- Applications of systems of linear equations

Background

Section 5.6 Systems of Linear Equations

The solutions of a linear equation in two variables, x and y are pairs of numbers. For any given x -value, we can solve for y to find a corresponding y -value and vice versa.

SYSTEMS OF EQUATIONS

A system of equations is a set of two or more equations. A solution to a system of equations is a set of values for the variables that makes all of the equations true.

There are three different ways to solve a system; graphing, substitution, and elimination. We are going to focus on substitution and elimination. We will talk about graphing a little bit too.

Example

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When we talk about solving a system, we are finding values for the variables that make both equations have the same solution. The first method is substitution. One form of substitution is as follows:

$$y = 14 - 3x$$
$$y = 11 - 2x$$

$$\begin{array}{r} 14 - 3x = 11 - 2x \\ +2x \quad +2x \\ \hline 14 - 1x = 11 \\ -14 \quad -14 \\ \hline -1x = -3 \\ \frac{-1x}{-1} = \frac{-3}{-1} \end{array}$$

$$x = 3$$

$$(3, 5)$$

$$y = 14 - 3(3)$$

$$y = 14 - 9$$
$$y = 5$$

$$y = 11 - 2(3)$$
$$y = 11 - 6$$
$$y = 5$$

Example

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Here is another situation where substitution is useful

$$\begin{aligned} y &= 5 - 2x \\ 3x + 2y &= 8 \end{aligned}$$

$$3x + 2(5 - 2x) = 8$$

$$3x + 10 - 4x = 8$$

$$-1x + 10 = 8$$
$$\quad \quad -10 \quad -10$$

$$-1x = -2$$
$$\quad \quad -1 \quad \quad -1$$

$$x = 2$$

$$(2, 1)$$

$$y = 5 - 2(2)$$

$$y = 5 - 4$$

$$y = 1$$

$$3(2) + 2y = 8$$

$$6 + 2y = 8$$

$$-6 \quad \quad -6$$

$$2y = 2$$

$$y = 1$$

Example

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Here is another situation where substitution is useful

$$\begin{array}{r} 3x - 7y = -11 \\ x + 4y = 9 \\ \quad -4y \quad -4y \\ \hline x = 9 - 4y \end{array}$$

$$\begin{array}{r} 3(9 - 4y) - 7y = -11 \\ 27 - 12y - 7y = -11 \\ \quad \quad \quad \checkmark \\ 27 - 19y = -11 \\ \underline{-27} \quad \underline{-27} \\ -19y = -38 \\ \underline{-19} \quad \underline{-19} \\ y = 2 \end{array}$$

$$(1, 2)$$

$$\begin{array}{r} 3x - 7(2) = -11 \\ 3x - 14 = -11 \\ \quad +14 \quad +14 \\ \hline 3x = 3 \end{array}$$

$$x = 1$$

$$\begin{array}{r} x + 4(2) = 9 \\ x + 8 = 9 \\ \quad -8 \quad -8 \\ \hline x = 1 \end{array}$$

$$x = 1$$

Warm-Up

Section 5.6 Systems of Linear Equations

Solve the system

$$y = 5x - 9$$

$$y = -2x + 5$$

$$\begin{aligned} 5x - 9 &= -2x + 5 \\ 5x + 9 &\quad + 9 \\ 5x &= -2x + 14 \\ + 2x &\quad + 2x \\ 7x &= 14 \\ 7x &\quad 7(x) \quad (2, 1) \end{aligned}$$

$$\begin{aligned} 5(2) - 9 \\ y &= 10 - 9 \\ y &= 1 \end{aligned}$$

Rules

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METHOD OF SUBSTITUTION

- Solve for one of the variables in one of the equations
- Substitute into the other equation to get an equation in one variable
- Solve the new equation, then find the value of the other variable by substituting back into one of the original equations.

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Background

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Substitution was one method that allowed us to find out where both equations had the same solution, however there is another method known as elimination which allows us to solve for the same solution as well.

When dealing with elimination there are three things that have to happen.

In order to eliminate:

1. you need to have the same variables in the equation
2. the coefficient attached as to be the same for both equations
3. one of the coefficients has to be positive and one of the coefficients has to be negative.

Example

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Let's try elimination.

$$3x + 5y = 1$$

$$2x - 5y = 9$$

What can be eliminated?

The 5y can be eliminated from both equations because they follow all of the rules. Once you eliminate the 5y's all you have to do is combine like terms, balance and solve, then substitute your solution into the original equations.

$$\begin{array}{r} 3x + \cancel{5y} = 1 \\ 2x - \cancel{5y} = 9 \\ \hline \frac{5x}{5} = \frac{10}{5} \\ x = 2 \end{array}$$

$$3(2) + 5y = 1$$

$$6 + 5y = 1$$

$$5y = -5$$

$$y = -1$$

$$2(2) - 5y = 9$$

$$4 - 5y = 9$$

$$-5y = 5$$

$$y = -1$$

Solution (2, -1)

Example

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$$5y - 3x = 21$$

$$7y - 3x = 27$$

With these equations they follow almost all of the rules, the only thing we would have to change is one of the $3x$ to a positive. We can do that by multiplying one of the entire equations by -1 .

$(5y - 3x = 21) \cdot -1$	$5(3) - 3x = 21$	$7(3) - 3x = 27$
$7y - 3x = 27$	$15 - 3x = 21$	$21 - 3x = 27$
$-5y + 3x = -21$	$-3x = 6$	$-3x = 6$
<hr/>	$x = -2$	$x = -2$
$\frac{2y}{2} = \frac{6}{2}$		

$$y = 3$$

Solution: $(-2, 3)$

Example

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$$4x + 5y = 35$$

$$8x - 6y = 22$$

For this situation we can change the x-value of the first equation to eliminate the x - terms. We can do this by multiplying the entire first equation by -2.

$$(4x + 5y = 35) (-2)$$

$$8x - 6y = 22$$

$$\underline{-8x - 10y = -70}$$

$$\underline{-16y = -48}$$

$$\underline{-16} \quad \underline{-16}$$

$$y = 3$$

$$4x + 5(3) = 35$$

$$4x + 15 = 35$$

$$4x = 20$$

$$x = 5$$

$$8x - 6(3) = 22$$

$$8x - 18 = 22$$

$$8x = 40$$

$$x = 5$$

Solution: (5, 3)

Example

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$$4x - 9y = -17$$

$$3x - 17y = -23$$

For this example, both equations will have to be changed, since the x - terms can both be change to 12 we will change them using multiplication.

$$(4x - 9y = -17)(3)$$

$$(3x - 17y = -23)(-4)$$

$$12x - 27y = -51$$

$$\underline{-12x + 68y = 92}$$

$$41y = 41$$

$$\frac{41y}{41} = \frac{41}{41}$$

$$y = 1$$

$$4x - 9(1) = -17$$

$$4x - 9 = -17$$

$$4x = -8$$

$$x = -2$$

$$3x - 17(1) = -23$$

$$3x - 17 = -23$$

$$3x = -6$$

$$x = -2$$

Solution: (-2, 1)

Rules

Section 5.6 Systems of Linear Equations

METHOD OF ELIMINATION

- Multiply one or both equations by constants so that the coefficients of one of the variables are either equal or are negatives of each other
- Add or subtract the equations to eliminate that variable and solve the resulting equation for the other variable
- Substitute back into one of the original equations to find the value of the first variable.

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Discussion

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There are often times when dealing with systems of equations where there will be no solution, one solution, or infinitely many solutions.

We have already looked at situations that have one solution, the following examples will show you what no solution will look like and what infinite solutions will look like.

Example

Section 5.6 Systems of Linear Equations

Let's use what we know to solve the following equations.

$$\begin{array}{rcl}
 2x + 3y = 7 & (2x + 3y = 7)(-1) & 2x + 3y = 8 \\
 2x + 3y = 8 & 2x + 3y = 8 & -2x - 3y = -7 \\
 \hline
 -2x - 3y = -7 & & \\
 \hline
 0 = 1 & &
 \end{array}$$

These are parallel lines so there is no solution.

$$\begin{array}{rcl}
 2(3x - 4y = 5) & (-4y + 3x = 5)(2) & \\
 8y - 6x = 7 & 8y - 6x = 7 & \\
 -8y + 6x = 10 & -8y + 6x = 10 & \\
 \hline
 0 = 17 & &
 \end{array}$$

Example

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Let's use what we know to solve the following equations.

If we rewrite them to align the x's and y's, we will see what happens in this situation.

$$5x + 2y = 14$$

$$2y + 5x = 14$$

$$\begin{array}{r} (5x + 2y = 14) - 1 \\ 5x + 2y = 14 \\ -5x - 2y = -14 \\ \hline 0 = 0 \end{array}$$

$$4x - y = 20$$

$$2y - 8x = -40$$

$$\begin{array}{r} 2(-y + 4x = 20) \\ -2y + 8x = 40 \end{array}$$

$$\begin{array}{rcl} (4x - y = 20)(2) & 8x - 2y = 40 \\ -8x + 2y = -40 & -8x + 2y = -40 \end{array}$$

Everything eliminates in both equations so there are infinite solutions because they are the same equations.

Example

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One rental car company charges \$20 plus \$0.10 per mile.

Another charges \$30 plus \$0.05 per mile. At what distance is the second company cheaper than the first?

$$y = 20 + 0.10x \quad 20 + 0.10(199) = 39.90$$

$$y = 30 + 0.05x \quad 30 + 0.05(199) = 39.95$$

Pick a method to solve to find the solution

$$\begin{array}{r} 20 + 0.10x = 30 + 0.05x \\ -20 \quad -20 \\ \hline 0.10x = 10 + 0.05x \\ -0.05x \quad -0.05x \\ \hline 0.05x = 10 \\ \cdot 0.05 \quad \cdot 0.05 \\ \hline x = 200 \end{array}$$

$$\begin{array}{l} 20 + (0.10)(201) = 40.10 \\ 30 + (0.05)(201) = 40.05 \end{array}$$

$$20 + 0.10(200) = 40$$

Graphing

Section 5.6 Systems of Linear Equations

Graphing systems of equations is another method that we use. When we graph systems we graph both equations and find the intersection point of the two graphs, however the only downfall to this method is sometimes the intersection point is not exact and is very difficult to find to an exact point.

If we graph two equations and they are parallel to each other, they will have no solution.

If we graph two equations and they end up on top of each other, they will have an infinite amount of solutions.

If we graph two equations and they cross each other once, they will have one solution.

Homework

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