Algebra 1

Unit 7

Quadratic Models

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Table of Contents**

|  |  |
| --- | --- |
| Title Page | Page 1 |
| Table of Contents | Page 2 |
| Objectives | Page 3 |
| Lesson 1 | Page 4 |
| Lesson 2 | Page 6 |
| Lesson 3 | Page 9 |
| Lesson 4 | Page 12 |
| Lesson 5 | Page 20 |
| Lesson 6 | Page 24 |

**Objectives**

By the end of this unit I will be able to….

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| 1. Understand the three forms of the quadratic equation: Vertex Form, Factored From, and General Form | 1. Solve for the quadratic equation using factoring, taking square roots, completing the square, and using the quadratic formula |
| 1. Sketch quadratic graphs and describe key features of those graphs. | 1. Write equations in equivalent forms to solve problems. |

**Lesson 1 – Quadratic Word Problems**

Objective:

I will explore quadratic models through word problems and be able to solve for different input and output situations.

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| Quadratic Models | When you throw a ball straight up into the air, its **height depends on three major factors, its starting position, the velocity at which it leaves your hand, and the force of gravity.** Earth's gravity causes objects to accelerate downward, gathering speed every second.  The acceleration due to gravity, called **g**, is **32 ft/s2.** It means that the object's downward speed increases 32 ft/s for each second in flight.  When this situation is graphed you end up plotting the height of the ball at each instant of time, the graph of the data is a **parabola**. |
| Quadratics and their Equations  Solving Quadratics symbolically | A baseball batter pops a ball straight up. The ball reaches a height of 68 feet before falling back down. Roughly 4 seconds after it is hit, the ball bounces off home plate. Sketch a graph that models the ball’s height in feet during its flight time in seconds. When is the ball 68 feet high? How many times will it be 20 feet hight? What are those times (approximate).    A model rocket blasts off and its engine shut down when it is 25 meters above the ground. Its velocity at the time is 50 m/s. Assume that it travels straight up and that the only force acting on it is the downward pull of gravity. In the metric system, the acceleration due to gravity is 9.8 m/s squared. The quadratic function h(t) = ½(-9.8)t² + 50t + 25 describes the rocket’s Projectile Motion.   1. Define the function variables and their units of measure for this situation. 2. What is the real world meaning of h(0) = 25? 3. How is the acceleration due to gravity, or g represented in the equation? How does it show that the force is downward? 4. Find h(x) when time is 11 seconds.   When asked to solve a quadratic symbolically, you need to leave a square root in your answer.  5(x + 2)² - 10 = 47 2(x + 1)² - 4 = 10  4 = -2(x – 3)² + 4 |

**Lesson 2 – Finding Roots and the Vertex Form**

Objective:

I will develop my understanding of various vocabulary terms that are used in describing quadratics...

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| Key terms for Quadratics  Vertex Form | Step 1: Using the key terms box, try to describe the three graphs  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_    Key terms to use  Minimum  Maximum  Roots  X-intercepts  Zeros  Quadratic  Parabola  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_    \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_    Step 2: Share what you wrote with your table partner  Step 3: summarize what your partner said about one of the graphs \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  There are a few different ways to write the equation to any quadratic function. One way is called the vertex form. The vertex form is useful when we want to graph a quadratic, because the vertex is found in the equation. When we graphed last unit, which is how we were able to graph. That form is: |
| Graphing Quadratics | Our goal for graphing quadratics is to find the maximum or the minumum, find any roots (X-intercepts), and to sketch graphs.  Let’s find the vertex (max or min), any x-intercepts, and graph the functions.    G(x) = (x – 5)²- 7  F(x) = (x + 2)²  Practice: Sketch each function, find any x-intercepts, and the min or max  H(x) = (x – 6)²+ 3  F(x) = x² - 2      H(x) = - (x + 4)² - 3    G(x) = -(x – 1)² + 6 |

**Lesson 3 – Vertex to General Form**

Objective:

I will convert from vertex form to general form and visa-versa.

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| Quadratic Forms | Vertex From f(x) = a(x – h)² + k  This gives you information about transformations of the parent function f(x) = x²  General Form f(x) = ax² + bx + c  This form helps us look at projectile motion situations |
| Breaking down the quadratic equation | The general fofrm f(x) = ax² + bx + c is the sum of three terms  A term is an algebraic expression that represents only multiplication and division between variables and constants.  A Sum of terms with nonnegative interger exponents is called a POLYNOMIAL.  Examples of Polynomials  17x 4.7x² + 3x  Reminder: when you simplify things, you can only combine like terms!    When you see the following expressions, what do you notice about each one?  3 + 4 10 – 3 2 + 5 12 – 5 20 – 13  These expressions are known as \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, because \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  We are going to use a couple of expressions to model squaring binomials with rectangle diagrams.        This diagram shows how to express 7² as (3 + 4)². We want to find the area of each of the inner rectangles. What is the sum of the rectangular area?  For the following expressions, draw a diagram and label the area of each rectangle and find the total area of each one.  A). (5 + 3)² B). (4+ 2)²  We can do the same when we subtract  A). (5 – 2)² B). (7 – 3)²  And we can do the same with variables  A). (x + 5)² B). (x – 3)² C). (x + 11)²  What about (x + 3)(x + 5)?  (x – 6)(x + 3)  Now, (x + 3)² + 4  2(x + 5)² - 8 |

**Lesson 4 – Factored Form**

Objective:

I will use roots to write equations in factored form.

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| Brainstorm | In the previous lessons we did a multiplication of binomials, and our result was always a trinomial.  What do you think we would have to do in order to take a trinomial and put it back into binomial form?  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| Changing from Trinomial Form to Binomial Form | Look at the following trinomial; we will use what we already know to change it to binomial form. |
| Try it |  |
| Perfect Squares  Identify perfect squares | Numbers like 49 are called **PERFECT SQUARES** because they are squares of integers, in this case  7 and -7.  The trinomial is the square of \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, so it is also a perfect square.  Which of the following trinomials are perfect squares? |
| Identify the roots or  x-intercepts of the equation | The roots of a quadratic equation are also known as the x-intercepts. This means that when you graph a quadratic equation, wherever your graph intersects the x-axis is a root or x-intercept.  Look at the following graph, what would be the root(s) or x-intercept(s)?    Root(s)/x-intercepts:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Now let’s take the same equation and work it backwards into binomial form.  What do you notice about the x-intercepts/roots and the binomial form? |
| Factored Form | The cool thing about having our trinomial written in binomial form is that it is an easy way to find the x-intercepts of our quadratic equation. When out quadratic equation is written in binomial form it is called **factored form**.  Factored form relies on the factors of our quadratic.  Let’s find the x-intercepts and the quadratic form of the following binomials.   1. 3. 2. 4. 3. 6. |
| DAY 2  Objective: | I will learn methods for writing equations in factored form that aren’t written as a perfect square. |
| Review from Day 1 | When we change our quadratic equations to binomial form we are creating an equation that is known as factored form. Instead of calling these equations binomial form, we will now call them factored form.  Write the quadratic equation in factored form.  What do you notice about the relationship between the quadratic form and the factored form?  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| Finding Factors of the Quadratic Form | Look at the following equation, can you find the factored form of this equation?  Write down what you think you would do.  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Check to see if your method will work.  Share your idea with your table group.  Let’s try one as a whole group.  Factors of \_\_\_\_\_\_\_\_\_\_\_  Middle term: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Which factors create a sum that is the middle term? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Factored Form: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Roots/x-intercepts: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| Try It | Factors of \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Middle term:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Which factors create a sum that is the middle term? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Factored Form: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Roots/x-intercepts: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Factors of \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Middle term:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Which factors create a sum that is the middle term? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Factored Form: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Roots/x-intercepts: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Factors of \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Middle term:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Which factors create a sum that is the middle term? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Factored Form: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Roots/x-intercepts: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| Day 3: Tying it all together  Objective: | I will use everything I have learned to work on finding x-intercepts of quadratics and write equations in quadratic form from factored form and vertex form. |
| Finding Roots of a Factored Form Equation |  |
| Converting from Factored Form to General Form |  |
| Special Cases  of Factored Form |  |
| Converting from Vertex Form to General Form | How are we going to take this equation that is in vertex form and convert it to general form?  Vertex: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| Practice | Vertex: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Vertex: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Vertex: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

**Lesson 5 – Completing the Square**

Objective:

I will use what I know about factoring to help me understand how to complete a square. I will learn that completing the square is writing an equation in vertex form.

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| Factoring | We have been working on factoring a variety of equations. Are the following quadratics factorable? If so, factor them. |
| Is it Factorable? | Consider the equation. Can you factor it? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| Take a Deeper Look | If we use our calculator to find the decimal approximations for the solutions, what do you notice?  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| Trying the Process | Can we factor the following equation? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| The Keys to Completing the Square | The key to solving by completing the square is to express one side of the equation as a perfect-square trinomial. That problems that we have dealt with so far are equations in the form  , where the **leading coefficient** is 1. However there are other perfect-square trinomials. Look at the example.    However the perfect square is not always a possibility.  Add \_\_\_\_\_\_\_to both sides  Divide both sides by \_\_\_\_\_  Decide what number to add to  both sides to get a perfect-square  trinomial  Add \_\_\_\_\_\_\_\_\_\_ to both sides to  complete the square  Write the perfect-square trinomial  as a squared binomial and combine  any like terms  Take the square root of both sides  Add \_\_\_\_\_\_\_\_ to both sides  Solutions are: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| Another Challenge | To convert, complete the square.  Factor the 2 from the coefficients:    Rewrite the expression as a squared binomial : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Distribute the 2: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Combine like terms to get the vertex form: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  Find the x-intercepts: |

**Lesson 6 – Quadratic Formula**

Objective:

I will learn the quadratic formula which is another way to find the x-intercepts or roots...

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| The Methods We Have Used So Far |  |
| Quadratic Formula |  |